EYES AND SPECTACLES

DR. M. VON ROHR

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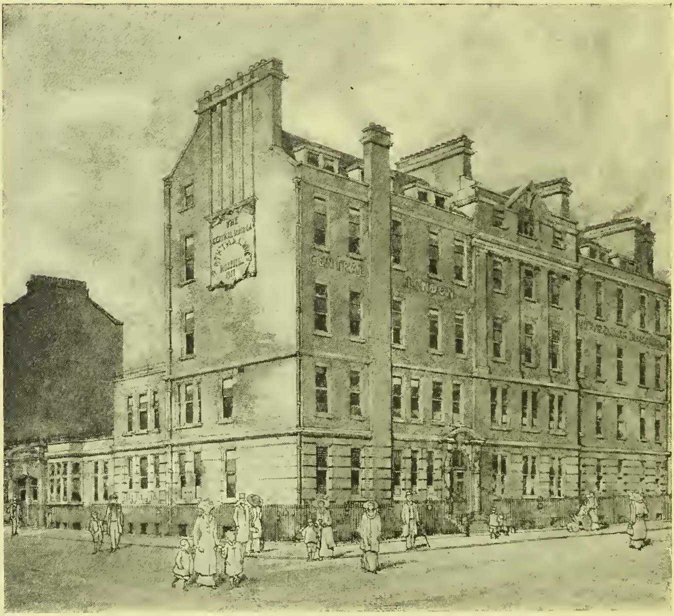

THE INSTITUTE OF OPHTHALMOLOGY
LONDON

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EYES AND SPECTACLES

BY

DR. M. VON ROHR.

Rendered into English

BY

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PREFACE.

An attempt has been made in the accompanying book to bring together the most important points about spectacles. In the modern science of spectacles the chief importance must be laid upon vision with moving eyes (looking about); it was therefore necessary to consider in the first place the eye itself, and also perspective as a form of perception by means of which the arrangement in space of the outer world became manifest to the observer. This is dealt with by the introduction of the perspective bundle of rays with its apex at the centre of rotation of the eye, and is further more clearly explained and illustrated by means of an assumed—generally plane—projection surface.

The second and most important section of the book deals with spectacle lenses, and in this the two problems which are so important in the modern theory of spectacles are dealt with, namely, the increased clearness of vision of ametropic eyes and the alteration of direction of the object perceived. In the single glass the increased clearness of vision is the most important, and this is followed through the different forms of anastigmatic (axially symmetrical), prismatic, and astigmatic lenses, in so far as the exactly reproducing lenses and systems bear upon this problem. In dealing with spectacles for the two eyes the alteration in direction is shown to be of considerable importance.

The final portion of the book deals with spectacle frames, which includes pictures of the various forms of pince-nez and spectacles as well as lorgnettes.

The employment of formulæ and of geometrical demonstrations has been avoided as far as possible, but they could not be entirely dispensed with if a sufficient insight into the subject was to be given. It is hoped that the employment of a large number of illustrations (mostly new) will make the matter easier to understand. If this book will contribute, however slightly, to the knowledge of the problems associated with spectacles it will have afforded sufficient reason for its appearance.

MORITZ VON ROHR.

JENA, 1912.

TRANSLATOR'S PREFACE.

The accompanying little book by Dr. Moritz von Rohr has been of considerable help to the translator, and he felt that it ought to be made accessible to English readers.

Dr. von Rohr is recognised as one of the great authorities of the present time on visual optics; and he has used the facilities of the scientific laboratories of Messrs. Zeiss, of Jena, to elucidate many of the problems which have to be solved in the process of evolution towards the perfect spectacle.

The book embodying these results should be read by those who have to do with the prescribing of lenses as well as those who make them; and they will find many difficulties explained and methods of avoiding them given.

The book is clearly written and very well illustrated. Many of the illustrations show great ingenuity, and all are well adapted to elucidate the subject under discussion.

The translation has had the advantage of Dr. von Rohr's own revision, and this is a guarantee that the English translation represents his views accurately. It is hoped, therefore, that this edition may prove of service.

A. H. LEVY.

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EYES AND SPECTACLES.

I. THE EYE AND ITS USE IN VISION.

THE eye is undoubtedly the most important of optical instruments. In the following pages we shall deal not only with the optical arrangements of the eye, but also with certain incidences of the act of vision; more especially with the quick and easy movement of each eye in its orbit, and with the yoked actions of the two eyes. In the discussion of these various points it will be seen how an instrument which, from the point of view of technical optics, is fairly imperfect, becomes capable of such extensive and accurate performances.

The Eye as a Fixed Optical System.

Light rays entering the eye first fall upon the cornea. This forms the anterior, more highly curved portion of the first coat which surrounds the whole eyeball. The cornea forms the front boundary of the anterior chamber which is filled with the aqueous humour. The iris is the back boundary of the anterior chamber. Immediately behind the iris is the crystalline lens, the surfaces of which in the normal eye, as will be mentioned later, can be altered in curvature.* Behind the lens lies the vitreous, a jelly-like mass which fills the greater

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* We must, however, remark that the crystalline of children and young people under twenty consists of extremely thin layers, the indices of which increase *gradually* towards the centre of the crystalline. C. Hess, the Würzburg ophthalmologist, has shown that for people over twenty this gradual increase no longer holds good, but that two surfaces exist where the index shows an abrupt increase. These two surfaces include between themselves a biconvex nuclear lens; whereas the two including lenses have the form of thin dispersive menisci. As the function of such a

part of the interior of the globe and reaches to the retina. The axial length from the summit of the cornea to the yellow spot depression is in a normal eye 24 mm. The retina is formed by the branchings and terminations (rods and cones) of the optic nerve, and is the layer which is sensitive to light. The ability to distinguish details varies greatly in different parts of the retina. It is most acute in the central portion, namely, at the yellow spot, and decreases towards the periphery. At the point where the trunk of the optic nerve enters the eye there is no light perception at all, and therefore this area is called the "blind spot."

The optical system of the eye is formed by the cornea, aqueous, and lens, and is, therefore, bounded by two different media, namely, in front by air, and behind by vitreous. Its two focal lengths are, therefore, not equal, and we must differentiate an anterior focal length (towards the air) from a posterior focal length (towards the vitreous).

A normal or emmetropic eye, in a state of rest, focuses the light rays from a distant object upon the retina. If the eyeball be too long (and in myopia an axial length up to 36 mm. may occur), then the rays will come to a focus in the vitreous before reaching the retina, and the eye is termed short-sighted or myopic. If, on the other hand, the eyeball be too short (and measurements as low as 21 mm. have been found), so that rays would only come to a focus behind the retina, the eye is termed longsighted (hypermetropic or hyperopic). The cause of these irregularities (ametropias) lies in an

system showing a varying index of refraction is rather difficult to understand, we shall introduce the term "total index." We obtain it in choosing a crystalline lens of the same outer curvatures, and giving to the optical system of the eye the same equivalent focus. Supposing the index of the cornea to be 1.376, the aqueous and vitreous 1.336, we obtain a total index of 1.4085 for a crystalline free from accommodation.

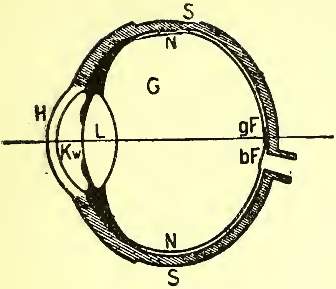


FIG. 1. A horizontal section through the right eye.

H—Cornea.
Kw—Aqueous.
G—Vitreous.
gF—Yellow Spot.
SS—Sclerotic.
L—Crystalline Lens.
NN—Retina.
*b*F—Blind Spot

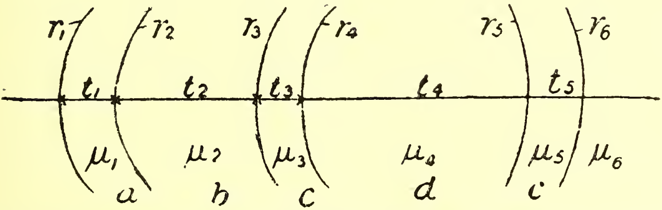


FIG. 2. Vertex curvatures and principal points in the optical system of Gullstrand's schematic resting eye.

a—Medium of the Cornea.
b—The Aqueous and the Vitreous.
c—The Lens.
d—The equivalent nuclear Lens.

RADII.

	At Rest. mm.	Accomd. mm.	mm.	mm.
$r_1 =$	7·7	7·7	$t_1 = 0·5$	$\mu_1 = 1·376$
$r_2 =$	6·8	6·8	$t_2 = 3·1$	$\mu_2 = 1·336$
$r_3 =$	10·0	5·33	$t_3 = 0·546$	$\mu_3 = 1·386$
$r_4 =$	7·911	2·655	$t_4 = 2·419$	$\mu_4 = 1·406$
$r_5 =$	−5·76	−2·655	$t_5 = 0·635$	$\mu_5 = 1·386$
$r_6 =$	−6·0	−5·33		$\mu_6 = 1·336$
$f =$	{ 17·06 and 22·785			

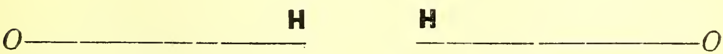


FIG. 3. The relation of the object points to the principal points. Both possible positions of the object *O* to the principal point of the eye H are given; to the left a real object point. to the right a virtual object point, both on the optic axis.

abnormal lengthening of the optic axis in myopia, and an abnormal shortening in hypermetropia (axial ametropia).

Assuming a normal eye in a position of rest, its anterior focus will be about 17.1 mm., and its posterior focus 22.8 mm. As will presently be shown, it is possible by simple mathematical methods to determine the size and position of an image formed by a system bounded by two different media, such as that of the eye.

For this purpose a *schematic eye* is assumed. This is a simplification of the natural eye, but the mathematical results agree with those which would be obtained from an ideally perfect natural eye. Data for the calculation of a schematic eye have been given by Listing and Helmholtz—but in these pages we shall use the most recent values as determined by A. Gullstrand, of Upsala. In order that the refractive power of the optical system, as well as the position of the principal points in a schematic eye should agree with the ideal natural eye, it was necessary to assume in the interior of the crystalline lens an equivalent nuclear lens. The dimensions and curvatures of the various surfaces are given in the accompanying fig. 2, and it will be seen that the two principal points (**H** and **H′**) are not far away from the cornea. The distances being for **H** 1.35 mm., and for **H′** 1.60 mm.

It is customary in optical calculations, when dealing with the position and size of images near the optic axis, to refer the calculations to the principal points; the reason for this will appear when considering accommodation. Thus the distance of an object point O is measured from the anterior (on the object side), principal point H, and is written

$$H\,O,$$

and is regarded as negative when it is measured

against the direction of the light, and as positive when in the same direction as the light. As it is customary to represent light as travelling from left to right, then H O will be negative when O lies to the left of H, as in the case of real objects, and H O will be positive when O lies to the right of H, as in the case of virtual objects.*

In the simpler calculations, however, the distances between the object point O and the anterior principal point H, and that between the image point O′, and the posterior principal point H′, are not used, but rather their reciprocal values, *i.e.*, Gullstrand's convergencies.† The unit of the latter is the reciprocal value of 1 meter, and is termed the Diopter (D). That is

$$1\ \text{D} = \frac{1}{1\ \text{m.}}$$

For instance, the reciprocal value of the focal length of the eye would be

$$\frac{1}{0.01706\ \text{m.}} = 58.64\ \text{D.}$$

and in general the reciprocal value of the focal length of a system is termed its refractive power R

$$\text{R} = \frac{1}{f}.$$

* There is unfortunately no generally accepted rule for the sign of a distance in optical calculations, and the rule followed here must, therefore, be considered as arbitrary. It is, however, observed not only by Abbe and his followers, but also by A. Gullstrand, and the importance of the latter author is great enough for his example to be followed.

† Convergency is used here in an entirely different sense from that when used in conjunction with vision by two eyes, when the "angle of convergence" indicates the alteration in the direction of the two visual axes.

Ophthalmologists therefore say that the refractive power of the optical system of a normal eye is 58.64 D. It might be asked why the value of the posterior focal length was not used for the determination of the refractive power, and the difficulty is explained by the following fact. If in an optical system distances occur in the denser medium, these are reduced to distances in air by dividing them by the refractive index of the medium. Convergencies are then found from the reduced length.

If one proceeds thus with the posterior focal length, which applies to the vitreous, one finds the reduced focal length to be

$$\frac{22.785 \text{ mm.}}{1.336} = 17.06 \text{ mm.},$$

and it will be seen that the value given for the refractive power of the system is identical with that obtained by the other method.

The above mentioned lengths are generally indicated by the small letters of the alphabet,

$$a = \text{H O}; \; b = \text{H}' \text{ O}',$$

and their convergencies by the corresponding capital letters; thus

$$\text{A} = \frac{1}{a}; \; \text{B} = \frac{1}{b}.$$

The formula which applies for reduced conjugate focal distances,

$$\frac{1}{b} = \frac{1}{a} + \frac{1}{f},$$

takes the form for the reduced convergencies of

$$\text{B} = \text{A} + \text{R}.$$

For the size of the image, if the object element a, vertical to the axis, corresponds to the image

element β, also vertical to the axis, the following proportion will hold :

$$a : b = \alpha : \beta ; \frac{\alpha}{a} = \frac{\beta}{b},$$

which can be written

$$\alpha \text{ A} = \beta \text{ B}.$$

As will be recognised from fig. 4, the following rule can be set up, since for small angles the angle can be expressed by the value of its tangent,

$$w = \alpha \text{ A} ; w' = \beta \text{ B},$$

therefore

$$w = w' ;$$

so that if the lengths in a denser medium of any system are reduced to air, then, for an object element

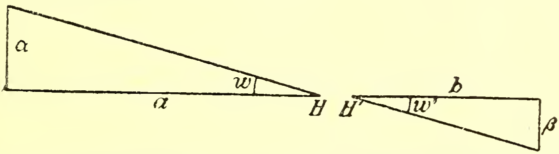


FIG. 4. The position and size of the image, and the angle w, w' between the principal axis and the ray for reduced conjugate distances.

α, the ray-axis angle w equals w', the ray-axis angle of the corresponding image element. Otherwise expressed, it means that for reduced distances the Gauss principal points are identical with Listing's nodal points. This observation is of advantage when the object is at a great distance, so that

$$\text{A} = 0$$

then

$$\text{B} = \text{R},$$

and it follows that

$$w = \beta \text{ R}$$

or

$$\beta = f\ w.$$

The size of the image in the posterior focal plane is the product of the focal length reduced to air, f, and the apparent size w of the distant object. This is a relation which will frequently be used in considering the size of the retinal image in the naked and in the spectacled eye.

These fundamental principles have to be applied as soon as one endeavours to determine the position of an axial point which can be seen clearly by an axially ametropic eye at rest. This point is termed the Far Point, **R.** It is postulated that this point can be determined by these approximate formulæ. It is to be noted that here, as elsewhere throughout this book, the spherical aberration of the optical system of the eye is neglected.

If then, as mentioned before, the axial length of the eye varies between 21 and 36 mm., let an example within these limits be calculated out. Assume an eye in which the foveal depression O′ lies 26.38 mm. behind the posterior principal point H′, which presumes the distance between the summit of the cornea and the posterior principal point to be that of an eye having almost exactly an axial length of 28 mm.

One obtains first the reduced distance b and the corresponding convergence B

$$b = \frac{0.02638 \text{ m.}}{1.336} = 0.01974 \text{ m.,}$$

$$\text{B} = 50.64 \text{ D.}$$

The fundamental equation obtains the form

$$\text{A} = \text{B} - \text{R},$$

and we find that

$$\text{B} = 50.64 \text{ D,}$$

$$- R = - 58.64 \text{ D}$$

$$A = - 8 \text{ D}$$

$$H\,O = a = - .125 \text{ m.}$$
$$= - 125 \text{ mm.}$$

The value A is the axial refraction of the axially ametropic eye and its corresponding distance a, as will be obvious from what was stated before, is measured from the anterior principal point H.

In this way the axial refraction of any other axial length can be calculated, and by this means one obtains the following

TABLE FOR THE LENGTHS l BETWEEN THE FOVEAL DEPRESSION AND THE VERTEX OF THE CORNEA, FOR THE CORRESPONDING AXIAL REFRACTIONS A.

A in D's.	l in mm.	A in D's.	l in mm.	A in D's.	l in mm.
10	21.07	0	24.38	−11	29.64
9	21.36	− 1	24.78	−12	30.24
8	21.65	− 2	25.19	−13	30.87
7	21.96	− 3	25.61	−14	31.53
6	22.27	− 4	26.05	−15	32.21
5	22.60	− 5	26.51	−16	32.94
4	22.92	− 6	26.98	−17	33.69
3	23.27	− 7	27.47	−18	34.48
2	23 63	− 8	27.98	−19	35.31
1	24.01	− 9	28.51	−20	36.18
		−10	29.07		

The graphic representation in fig. 5 has been constructed to give an easy conception of the relationship between the values l and A.

It was stated above that, in general, only eyes with axial ametropia would be considered, but here an exception must be made in favour of certain

curvature ametropias, as the case of eyes without lenses (aphakia) requires some discussion.

When the lens becomes opaque, as in cataract, light will once again enter the eye if the lens be removed. It is assumed that the operation has succeeded perfectly. In such an eye the refracting system consists of cornea alone, and therefore the total refractive power is much diminished,

$$R_h = 43.1 \text{ D},$$

and both principal points fall very near the corneal vertex. The focal lengths are correspondingly lengthened to

23.23 mm. in air and 31.03 mm. in the vitreous,

and thus a normal eye with an axial length of 24 mm. has become hypermetropic through removal of the lens. Later, on page 72, when considering the question of spectacles, it will be necessary to point out the consequences which ensue when such an eye is corrected by ordinary convex lenses.

In recent times, since the year 1890, following on the advice of the Austrian ophthalmologist, Fukala, the operation of removal of the lens has been practised in cases of high myopia. Assuming the optical system of such an eye to be normal, then after the operation the values given for refractive power and focal lengths will hold good, and it will be recognised that the refractive ametropia produced is rendered more or less harmless by the previously existing axial ametropia. By a simple calculation, neglecting spherical aberration, an axially myopic eye of about -25 D is, after the operation, found to see clearly at a distance. Eyes with a somewhat larger or smaller error will be corrected, as can also be found by simple calculation, by weak lenses. This matter will be dealt with more fully later on.

This operation is still performed; but some

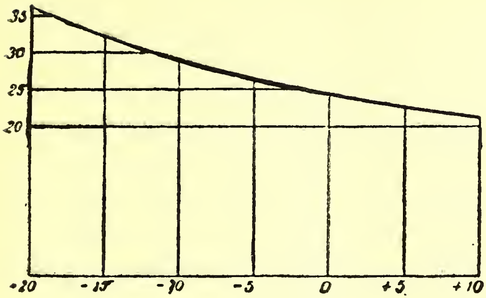
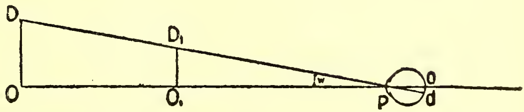


FIG. 5. The relation of axial refraction and length in axially ametropic eyes.

FIG. 6. Vision by means of the stationary eye.



w Inclination of principal ray to the optic axis or visual angle.
o d The arc on the retina corresponding to the visual angle *w*.

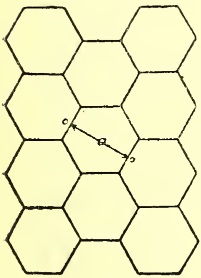


FIG. 7. The visual acuity of the human eye; *a* the average distance between the ends of two elements (cones).

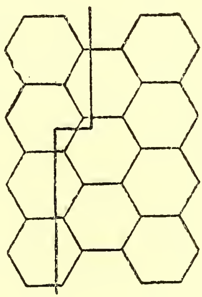
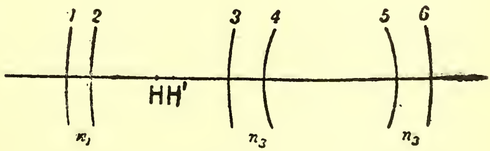


FIG. 8. The acuity of width perception.

FIG. 9. Vertex curvatures and principal points of the optical system of Gullstrand's schematic accommodating eye.



n_1—Cornea.
n_2—Aqueous and Vitreous.
n_3—Lens.
n_4—Equivalent nuclear lens.

surgeons have stated that it is not without certain dangers, and, if done at all, should only be done on one eye. This is not the place to go further into this question, but later on we shall have to speak of an optical method which has advantages over this operation, and which is free from the danger which, after all, attends all surgical procedures.

A healthy eye must be regarded as a centred system, since departures therefrom are so small as to render their consideration here unnecessary. The optic axis passes through the vertices of the lens, and the centre of the iris, to the macula. A true focal point, in the strict sense of the word (elimination of spherical aberration), never occurs, not even for rays coming from a point on the axis; but instead there are diffusion figures formed on the retina. These are of special interest when looking at bright points on a dark background. Under these circumstances, and as a result of the slight tension exerted upon the lens by its suspensory ligament, a four or eight-rayed star figure is seen, the physical theory of which has been worked out by Gullstrand. The reason why these diffusion figures do not interfere with vision more than they actually do, is partly because the iris, in bright illumination, contracts, and with the narrowing of the aperture of an optical system the diffusion figures are also diminished; and partly because the distribution of light is not uniform in these diffusion figures; that part which has most illumination, and therefore is more important from the point of view of visual acuity is only of small extent. The aberrations in oblique vision are, of course, greater, but these are not noticed on account of the great diminution of sensitiveness in the peripheral portions of the retina. The optical system of the eye is not corrected for chromatic aberration, and it shows the same errors in this respect as an ordinary convex lens. These colours

are not noticed, because the human eye is specially sensitive for only a small portion of the spectrum, namely, the yellow and the green, and therefore the blue and red coloured rings are generally ignored.

The entering cones of light are limited by the iris, which by its sphincter muscle is so opened and closed that its aperture is always circular and always symmetrical to the optic axis. This variation in the size of the aperture is chiefly for the purpose of regulating the quantity of light entering the eye. In bright illumination the aperture will contract down to a diameter of 2 mm., or less; in very feeble illumination it will dilate up to a diameter of 6 mm. and more. Later, another cause for the variation of the size of the pupil will be given, which cannot be dealt with here, as we are now concerned with the eye as an optical system which is rigid and at rest.

The iris is the only diaphragm in the eye, and there is nothing to stop down the diameter of the eye's pupil which is the image of the iris. The field of vision of the eye is very large, laterally it is only limited by the cheek and nose. The sensitiveness of the retina to light is also very widespread, so that even rays of light at right angles to the visual axis will still cause sensation. But, as already mentioned, the accuracy with which such a sensation is transmitted to the brain and projected into the outer world, diminishes very rapidly as soon as it is displaced from the yellow spot.

The inclination of every principal* ray w, on the object side, corresponds to a certain distance $o\ d$ of the image point d from the axial point o on the retina, and on the stimulation of such a point d, of

* In the optical papers of the Jena school principal rays are defined as those oblique rays which pass through the centre of the actual stop. As they are the barocentric lines of the solid oblique pencils limited by or passing through the stop, they are also to be considered as the axes of these solid oblique pencils.

which we do not become conscious but which transmits the sensation, the stimulus is sought for in the direction from which the principal ray enters the eye from the outside. An eye, rigid and at rest, can only judge directions from the centre P of the pupil, that is angular distance. Two objects O D and O_1 D_1 which, at the centre of the pupil will make the same angle w with the optic axis, will appear to be of similar size or, as it is generally put, will be of the same apparent size. These are measured by the trigonometrical tangents of the visual angle w, or by means of the following fraction

$$\text{Apparent size} = \frac{\text{Height}}{\text{Distance between object and pupil}}$$

If, without varying the distance, the height of the observed object be gradually decreased, or if the distance from an observed object be gradually increased, a point is ultimately reached at which the observed object will appear to have no height at all; it will then appear of indeterminate form—*i.e.*, a point. Should the object be formed of two points, these will be indistinguishable one from the other, and they will appear as a single point. The angle w thus obtained is termed the angular measure of visual acuity and its average value is taken as one minute of arc. The above mentioned fraction will then have the value

$$\tan 1' = \frac{\text{Height}}{\text{Distance}} = \frac{1}{3438},$$

which means that when objects are observed from a distance equal to 3,438 times their largest diameter, no details of their form will be apparent to an eye having normal vision, assuming that they are not specially well illuminated. For a shilling

with a diameter of 23 mm., this distance would be about 79 m. To this angular measure of visual acuity, as may be seen by fig. 7, corresponds the average distance *a* between two sensitive elements in the foveal depression, or the average diameter of a retinal element or cone, which, from the above given data about the focal length of the eye and the angular visual acuity, amounts to 0.00487 mm.,* and this is confirmed by anatomical investigations.

The Leipzig physiologist E. Hering has, however, pointed out that, assuming a mesh-work arrangement of the cone endings, as was demonstrated to be the case later by L. Heine, displacements of straight lines from one another can still be differentiated, even when far below the limit of average visual acuity, as will be seen from fig. 8. With such an arrangement it is possible to perceive whether one line be the continuation of the other even when the distance between the two stimulated rows of cones is below 4.9 μ. The perception of such displacements is made use of in an attachment for fine measuring instruments, the nonius or vernier—and some observers have been able to differentiate displacements having an angular value of only 10 seconds of arc. As an average value of the acuity of such width perception we will take half a minute of arc.

The above mentioned properties of an eye, as an optical system rigid and at rest, would apply almost equally as well to a dead eye removed from its orbit. That in such an eye the pupil does not vary according to the intensity of the illumination would not affect any of the above statements—since any such variation affects mostly the size and not very much the situation of the pupil. We shall

* If one takes with J. B. Listing a very small unit μ = 0.001 mm., then this value can be expressed as 4.87 μ.

now have to deal with those characteristics which differentiate the living from the dead eye.

The Accommodative Power of the Eye.

A normal eye possesses the power to increase the curvature of both surfaces of the lens and, by this means, to alter, within certain limits, the focal length of the optical system. As a result of measurements it has been found that the focal length can be reduced from 17.1 and 22.8 mm. to 14.2 and 18.9 mm. Man is thus in the position of being able to focus accurately, not simultaneously, but in rapid succession, objects at very varying distances. The ability to do this is termed the Power of Accommodation (power of adjustment). If the most distant point which the eye can see clearly when at rest or with relaxed accommodation, be termed the Far Point **R**, and the nearest point which the eye can see, using all accommodative power possible, the Near Point **P**, then in a normal eye* the far point will lie at infinity, and the near point at a finite distance in front of the cornea, which distance will increase with age from 10 cm. at twenty to 22 cm. at forty years of age. The distance beween Far and Near Points is termed the Range of Accommodation, and the difference between the reciprocals of these values, expressed in diopters, is termed the Amplitude of Accommodation.

With increasing age the power of accommodation decreases still further, and the near point gradually recedes from the eye, so that in previously normal-sighted people over fifty even the far point no longer lies at infinity, but obtains a positive distance (this is usually termed negative distance in English

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* In a short-sighted eye both these points are real and lie within infinity. In a far-sighted eye it may occur that no real point can be accurately focused on the retina, in which case both these points are virtual.

literature) from the eye. In other words, distant objects can only be seen clearly by such an eye by the use of accommodation or by the use of a convex lens to form an image at that distance behind the eye.

F. C. Donders has set up a table to show these variations in accommodation.*

Age in years.	Distance of the near point in cm.	Distance of the far point in cm.	Amplitude of accommodation in diopters
10	− 7.1	∞	14
15	− 8.3	∞	12
20	− 10	∞	10
25	− 11.8	∞	8.5
30	− 14.3	∞	7
35	− 18.2	∞	5.5
40	− 22.2	∞	4.5
45	− 28.6	∞	3.5
50	− 40	∞	2.5
55	− 66.6	400	1.75
60	− 200	200	1
65	400	133	0.5
70	100	80	0.25
75	57.1	57.1	0
80	40	40	0

The above described condition is termed Presbyopia (Old Sight), and what makes it specially troublesome is that near objects cannot be seen clearly without the help of glasses.

In the process of accommodation both surfaces of the lens become more strongly curved and the lens increases in thickness, the layers of the lens taking up different relative positions. The refractive power of the lens becomes greater, not only because the

* More recently Duane and also Clarke have given tables more accurate than that of Donders. (*Trans.*)

surfaces become more curved, but also because the total index becomes higher. In maximum accommodation a value of 1.4263 is reached. The great increase in refractive power by means of accommodation which enables, for instance, a person twenty years of age to see objects clearly at a distance of from 10 cm. to infinity, and which is brought about by a comparatively small change in the form of the lens, could only be obtained by means of the stratified structure of the lens, the peculiar effect of which has been discovered and made clear by Gullstrand's investigations.

The optical system under these conditions is represented in fig. 9, and on comparing this with fig. 2 it will be seen that certain changes have occurred. The focal lengths are now, as already mentioned, 14.17 mm. in air and 18.93 in the vitreous, and also we find to the total refractive power

$$R_{akk} = 70.57 \text{ D.}$$

Further, the principal points will also be found to be displaced backwards towards the interior of the eye to the extent of about 0.45 mm. To be exact, the distance from the vertex of the cornea is for **H** 1.77 mm., and for **H**′ 2.09 mm. It is of great advantage that this displacement is so slight even in the maximal accommodation of which the eye of a twenty-year-old person is capable, for in most physiological measurements a difference of ½ mm. is within the normal limits of error. It is permissible, therefore, to neglect the accommodative displacement, and to have regard generally to the position in relaxed accommodation. This, for all practical purposes, unchangeable position of the principal points is the reason why, when calculating the size and position of images in the eye, it is customary to proceed from the principal points and not from the focal points, which would be quite feasible.

In accommodation the iris is pushed forward. A change also occurs in the diameter of the pupil even if the intensity of the illumination be unaltered. It contracts on accommodating for near objects and dilates on fixing distant objects.

If this small displacement of the iris be neglected and, further, if it be assumed, for the sake of simplicity, that the centre of the pupil, in all stages of accommodation, coincides with the anterior principal point, it can be shown that in an eye capable of accommodation the size of the retinal image is proportional to the angle of inclination w of the principal ray. The apparent size of an object is therefore unaltered by the process of accommodation because the direction of those principal rays which determine the visual angle is not appreciably altered.

It might appear that a single eye, capable of accommodation, would be able to determine not only the direction of the principal ray, but also, from the intensity of the accommodative effort, the distance of the object. This is, however, only the case for objects which are very near. Manifestly the angular measure of visual acuity must be decisive for the exactness of the accommodation. As long as, with incorrect accommodation, the circles of diffusion are smaller than the 3,438th part of the distance between the pupil and the object fixed, so long will they not be noticed, and there will be no reason for any alteration of accommodation. It is self-evident that, since accommodation depends upon the alteration in form of the crystalline lens, when the lens is removed, as in operations for cataract, the ability to accommodate disappears.

The Eye in Direct Vision.

A much more important departure from the assumption of a resting eye, with which we started,

arises when we consider that in ordinary use the eye is being constantly moved in its socket.

When the attention of the observer is directed to a certain point, the eye is so moved that the image of this point will fall upon the macular depression. It is impossible to free oneself from this habit in ordinary use of the eye; and to attain this freedom for the purpose of physiological experiment, a certain amount of practice is necessary. This habit explains the fact that, ordinarily, one does not notice the low visual acuity of the peripheral parts of the retina, or the presence of the blind spot, etc. The light perception of the peripheral parts of the retina is chiefly used to give the eye a general idea of direction, in order that it may be rapidly directed towards the point which has attracted attention. The eyeball is moved by means of six different muscles, in such a manner that the globe is moved within its socket like the ball in a ball and socket joint. The centre about which this movement takes place, the Centre of Rotation, is situated about 13 mm. behind the cornea or 10.5 mm. behind the pupil. In the following pages, when dealing with wide-angled visual fields, the following terms are used. When vision is so carried out that the eye is directed to the various observed (fixed) points, it is called Direct Vision; but when the visual act is performed by a fixed (immobile) eye it is termed Indirect Vision. For an immobile eye can only fix a single point somewhere in the centre of the field, and the rest of the expanse of the field of vision must necessarily be seen only indirectly. Therefore, assuming a visual field of considerable angular width, indirect vision will mean seeing with an immobile eye, and direct vision seeing with a moving eye.

The two previously mentioned axial points, the Far Point and the Near Point, participate necessarily in this movement of the eyeball about its

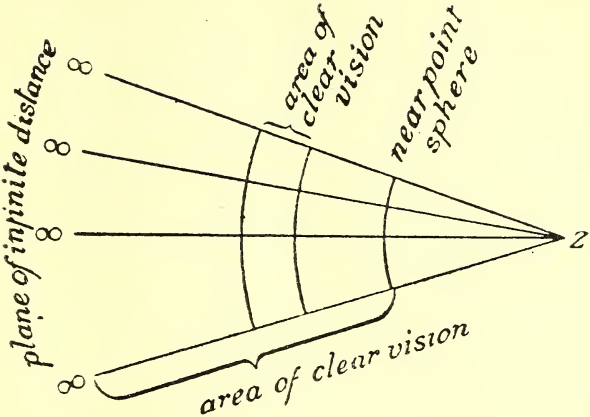


FIG. 10. Space and spheres of clear vision in an emmetropic eye.

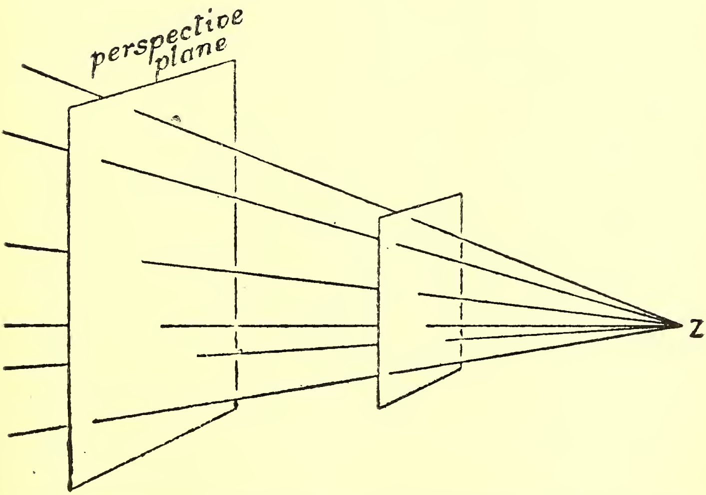


FIG. 11. A perspective pencil of rays, two vertical perspective planes and the similar plane perspectives arising on them.

centre of rotation. They move along the segment of a sphere, the radius of which is their distance from the rotation centre Z. In an emmetropic eye, whose far point, along the axis, lies at infinity, the far point sphere becomes a plane at infinity, while the near point sphere will naturally be real and will lie at a finite distance in front of the eye. The area of clear vision thus bounded will be occupied by concentric spheres of clear vision, each of which will correspond to a definite accommodative state. In a short-sighted eye these two boundaries of clear vision, the far as well as the near point segments, will be real, and will lie at a finite distance in front of the eye. In a far-sighted eye, the far point sphere will necessarily be virtual at a finite distance behind the eye, but no definite statement can be made as to the position of the near point segment in such an eye. According to the range of accommodation (age of the hyperope) the near point segment will be real at a finite distance in front of the eye, or at an infinite distance, or virtual, at a finite distance behind the eye.

We now have to deal shortly with the direction of the lines of vision, for upon this the *Perspective* of solid objects depends. It must be perfectly clearly understood that these directions of vision are entirely independent of the accuracy of visual perception. At the same point from a distant object, an ametropic eye will make exactly the same movements as an emmetropic one, although it will not see details clearly but only in diffusion figures. For the same reason, when speaking later of vision through spectacle lenses, the clearness of perception will be treated quite separately from the question of alteration of the line of vision.

The line joining a certain point with the centre of rotation of the eye is termed the line of vision or visual axis, and we will assume here that this corre-

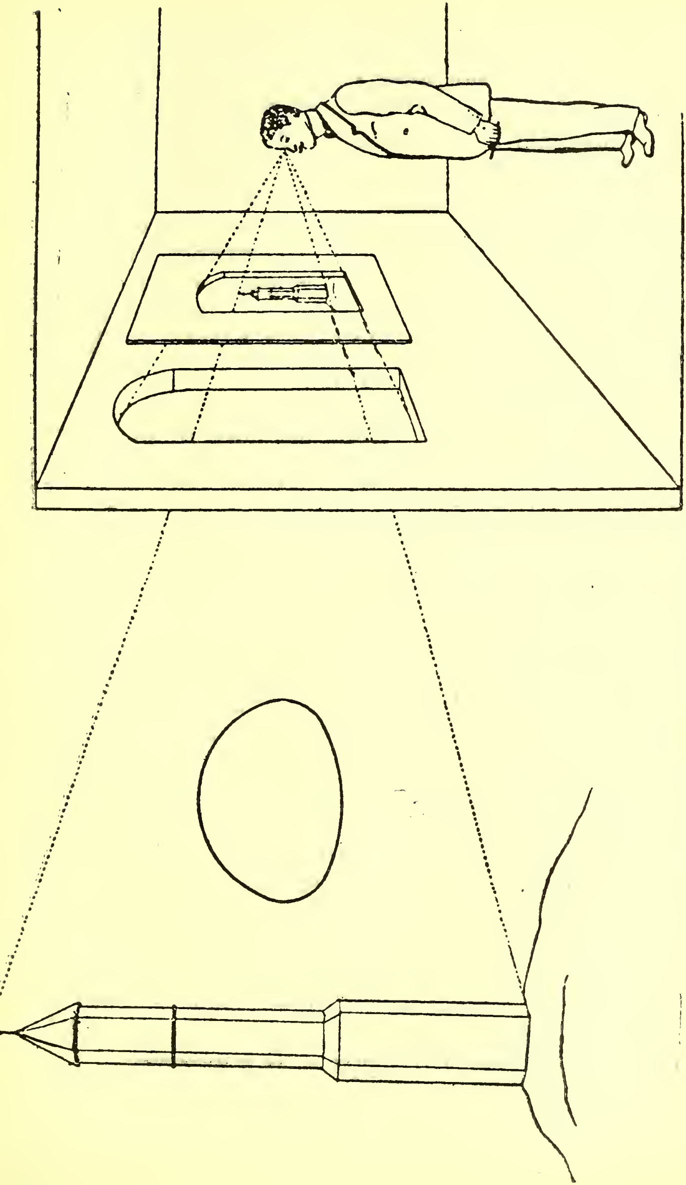


FIG. 12. A plane perspective as produced by an artist.

sponds with the optic axis when the point is fixed. Based upon this assumption, which is sufficiently accurate for the purposes of this discussion, there will be, if the head be not moved, one point common to all possible lines of vision, viz., the Centre of Rotation of the Eye. This point will therefore serve as the Perspective Centre in direct vision.

As to every point visible from Z there corresponds a definite line of vision, the whole surface will give rise to a definite bundle of rays with its apex at Z. It will be easily understood that only the points which, for Z, vary in width and height, or have different lateral distances, can form new lines of vision; while of two objects lying in the same direction of vision, but only varying in depth of distance, the near one will cover the more distant. The perspective pencil of rays is therefore spread out in all directions, upwards and downwards and to the right and left. If a section be taken through a perspective plane (fig. 11) every line of vision will correspond to a point of transmission in this plane, and all these points of transmission together form the plane of perspective of a solid object. Following its origin, the plane perspective from Z will correspond point for point with the surface of the solid object. In practice one places the perspective planes at right angles to a definite, as a rule horizontal, principal line of vision; for according to their distance from Z varying perspective planes are possible, but all plane perspectives arising upon them are strictly similar to each other. Each one of them regarded from the correct point Z will call forth the original perspective bundle of rays, and therefore will be able to replace the object, as far as the direction, in which each of its points will appear, is concerned. The distance of Z for each plane perspective is naturally different, and in general differs from the distance

of the object, but it has already been stated that judgment of the distance of objects by the intensity of the accommodative effort is very uncertain.

The practical application of these theoretical principles will be easily understood from a consideration of fig. 12. This illustration for the production of a plane perspective by an artist, was suggested by Leonardo da Vinci, and in our case we have pictured a number of points of transmission of the lines of vision. The meaning of the term "apparent size" of an object will also be obvious from this sketch, for from the standpoint of the drawer the large tower appears smaller than the window.

So far we have been dealing with the perspective of direct vision. As far as indirect vision is concerned, it can be stated that for the small field which can be seen clearly with a fixed eye the same principles will apply. Only the centre of these perspectives will be the centre of the pupil of the eye. Therefore, as far as clear vision occurs, for the small visual angle corresponding to the yellow spot the perspective of indirect vision will be determined by the pupil, while for the far more important direct vision the perspective centre will be the Centre of Rotation of the eye, which lies 10.5 mm. behind the pupil.

We arrive therefore at the conclusion, as will be seen from fig. 13, that for the perception of a solid object of considerable apparent size two perspective centres are in operation. One important, immovable, acting for all fixed points—the Centre of Rotation of the eye—and another unimportant, movable—the moving pupil of the eye, acting for the neighbourhood of each fixed point. These filling-in perspectives overlap at the edges without, however, interfering with the clearness of perception, because all the more important, *i.e.* fixed, points are determined by the lines of vision. The reasons why these rather

complicated relations do not interfere more in ordinary vision are various. For one thing, we are not practised in indirect vision, and also, for objects at some distance, the difference of 1 cm., the distance separating the two centres, is too small to produce any noticeable effect. For this reason one cannot assign to the difference between these two perspectives, the principal and the filling-in, any considerable importance in judging depth with one eye.

The fact that the eye is accustomed to move rapidly about its centre of rotation has the result that, in direct vision, there is no direction specially preferred like that of the axis in an optical instrument. The construction of plane perspectives for the moving eye is therefore rather arbitrary, because the direction of the perspective planes is indeterminate. For practical purposes we assume a middle, generally horizontal, direction, and erect thereon a (consequently vertical) perspective plane.

The Eye and Plane Perspective.

The problem of reproducing naturally upon a flat surface by means of pencil or brush that which is seen by the eye, arose very early in history. Gradually, as a solution of this problem, the doctrine of central projection or perspective came into being. In the terms already used this problem may be expressed as follows. It is required to reproduce upon the plane drawing surface the figure formed by the points of transmission of the perspective bundle of rays. We will then have on this plane surface the plane perspective due to direct vision. As a result of this perspective relationship the plane perspective can be introduced at a certain point (depending upon the scale) between the object and the centre of rotation of the eye; and if the distance be not less than the near point it will then

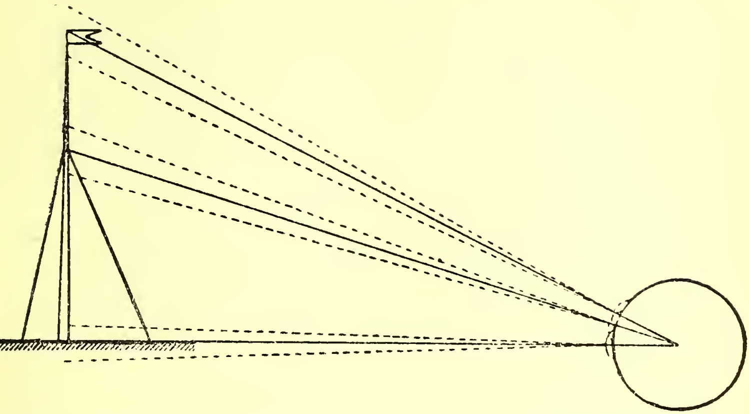


FIG. 13. Schematic representation of the principal and the filling-in perspectives. The eye (drawn on much too large a scale) is represented in three positions accordingly as it is fixing the base, middle and top of the flagstaff. The apparent size of these three positions will be judged by the angle of rotation of the eye about its centre, in direct vision. The parts in the neighbourhood of these points, which are less important to the observer, are viewed by the respective filling-in perspective represented by the dotted lines, and it is uncertain how far these extend.

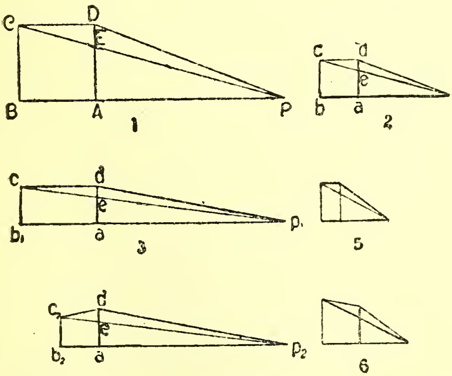


FIG. 14. The influence of the visual angle on the interpretation of perspective drawings.

1.—Projection in constructing a plane perspective. 2.—Viewing a reduced copy from the correct distance. 3 [5] — The deepening [flattening] of a drawing, recognised as right-angled, as the result of too great [too small] a distance from the eye. 4 [6]—The reduction [enlargement] of the background as the result of too great [too small] a distance from the eye, knowing the depth.

completely represent the object to the observing single eye in direct vision, as far as the apparent size is concerned. The filling-in perspectives differ somewhat from the natural ones, but, as has been stated above, they are of little importance. It is only necessary to state that this one-sided consideration of direct vision suffices; good pictures will furnish sufficient examples.

On account of the whole origin of plane perspectives, it is essentially necessary to retain the proper distance between the picture and the centre of rotation of the eye, if the same impressions are to be produced by observing the picture as by observing the object itself. We speak in this sense of the correct standpoint or position. Larger lateral deviations from this position are generally avoided, because pictures are so drawn that one has to view them from a point about opposite the middle of the picture, but the distance from the picture is frequently incorrect. The results of this error of position have been studied long ago, in the second half of the 18th century, by J. H. Lambert.

In fig. 14 we have assumed a perspective construction of one of the faces of a cube. In 1 the perspective centre P is assumed to lie on the prolongation of the line B A. The perspective or screen plane passes through A D so that E represents the point C. If now, as in 2, a reduced (say half-scale) copy $d\ e\ a$ of the plane perspective be so regarded, that the centre of rotation of the eye p is at such a distance from the vertical line $d\ a$, that $a\ p = \frac{1}{2}$ A P, then the angles $d\ p\ a$ and $e\ p\ a$ in the corresponding perspective processes will be equal. If one knows that $d\ e\ a$ is the perspective representation of a right-angled figure, then the constructions of the parallels $d\ c$ and $a\ b$, as well as the prolongation of $p\ e$ to c after letting fall the line $c\ b$, will produce a square, because the whole fig. 2 is exactly

similar to 1. The same conclusion would be reached if one knew that, in a figure bounded by two verticals, the depth $a\ b$ equalled the height $d\ a$. It would then be possible to deduce from similar reasons that the point c vertically above b would lie upon a line passing through d and parallel to the base.

Let it, however, be assumed that, as in 3, the centre of rotation of the eye p_1 is at too great a distance from the copy $a\ e\ d$ of the plane perspective—so that the visual angles determined by these points $d\ p_1\ a$ and $e\ p_1\ a$ are too small. Then, we have the same two possibilities of a judgment of space based upon our experience. If the height (assumed to be always the same) of the fig. A B C D be known (say that of a wall), then one will construct, as in 3, by means of the parallels through d and the prolongation of $p_1\ e$ the figure $a\ b_1\ c\ d$, that is a right-angled figure with a depth $a\ b_1$, which is too great in the same proportion as the actual distance $a\ p_1$ is greater than the correct distance $a\ p$. The other possibility is that from experience we are more familiar with the depth $a\ d = a\ b_2$ then a figure $a\ b_2\ c_2\ d$, as in 4, will be constructed, in which the more distant objects $c_2\ b_2$ will appear too small. Exactly the opposite effects will be produced if the visual angles be too great (viewing distance too small), as in 5 and 6.

It can obviously make no difference to the conclusions reached, whether the plane perspective under observation has a physical existence or not, whether it really exists upon a drawn or painted surface, or is merely produced by appropriate instruments for subjective use, where the image is virtual. The deduction of the results above obtained depends entirely upon the various alterations of the visual angles with which the plane perspective, or a copy of it, is presented to the eye.

To sum up, the result of any alteration of the

correct visual angle (or maintaining an incorrect viewing distance of a physical perspective) may be stated as follows: If, when viewing a correctly constructed perspective of a three dimensional object, the visual angle be diminished (the distance too great), then the conditions will be such as will tend to a false interpretation of the perspective, and this false interpretation will vary according to the experience of the observer, either the whole relief of the picture will be deepened, or the background will appear diminished. If the visual angle be increased (viewing distance too small), then the opposite errors may occur.

Vision with Both Eyes.

As a rule we have to deal with the simultaneous use of both eyes. In this case an object, or part of an object, is *fixed, i.e.* both eyes are so turned by their individual six muscles that the fixed point is focused upon both foveal depressions. The difference in the direction of the two visual lines is termed the Angle of Convergence. Although both eyes are stimulated, yet the fixed object is seen singly. Other points, the eyes being kept quiet, will only appear singly when they fall upon such points of the two retinæ as bear a similar relation, as regards position, to their corresponding foveal depressions. The plane which contains all singly-appearing object points is termed the Horopter. All object points not falling within the horopter will cause double vision, because their images will fall upon dissimilar (non-corresponding) retinal areas. These double images generally escape the attention of eyes not practised in these experiments. The character of these double images varies according as to whether the object producing them lies nearer or further than the fixation point. By this means we are able to estimate the dimensions of depth with-

out any movement of the eyes. Such a case would arise when objects are only momentarily illuminated as, for instance, the illumination of a night landscape by a flash of lightning.

Excepting conditions which occur very rarely, the method employed in judging depth (relative distance of objects) is to fix rapidly, one after the other, the various objects in space, and for this purpose it is necessary continually to alter the convergence of the optic axes. The regulation of the action of the eye muscles is extremely fine, the requirement, namely, to fix an object rapidly with the two eyes, means, speaking geometrically, to move the eyes rapidly so that both visual lines towards the object shall remain in one plane, that, namely, which will include both centres of rotation and the point fixed. With any change in the convergence there is associated, in the normal eye, a change in the accommodation, in the sense that with increased convergence the curvatures of both surfaces of the lens become increased. It is possible, however, by practice to overcome this habitual coordination.

The eyes, however, cannot well be used for the absolute measurement of distance, because the sensitiveness to the movements of the ocular muscles is not sufficiently fine to determine the very small angles of convergence, which would be produced by objects at any considerable distance away, having regard to the very small base line (the distance between the two centres of rotation varies in different individuals from 50 to 72 mm.). If, however, we have two objects, of which one is fixed and the other is seen in approximately the same direction, it is possible by means of the accurate judgment of width (p. 15) to estimate fairly accurately any existing difference of distance. Bearing in mind the above-mentioned distance between the centres of

rotation, and assuming an average acuity for the perception of width of half a minute of arc, then the distance *a* from the observer to an object which can be seen to stand out from an infinitely distant background, will lie between 350 and 500 m. This distance *a* is termed the limit of perception of depth in free vision. If the same expressions be used as when seeing with one eye, then seeing with both eyes will have this peculiarity. An object of three dimensions viewed with moving eyes will present a different perspective to each eye, the perspective centre of which will be the corresponding centre of rotation of that eye. The fusion of these two perspectives, differing in a definite way from each other, is brought about by referring the different stimuli to the outer world, and not by optical means. Indeed, it is an essential condition, which is automatically fulfilled in natural vision, that there shall be two different images, one for each eye.

II. SPECTACLE LENSES.

AMONG the various abnormalities which can be corrected by lenses, we shall deal first with the axial ametropias of symmetrical eyes, and therefore have to consider first

SYMMETRICAL LENSES.

The Combination of the Spectacle Lens and the Axially Ametropic Resting Eye.

It is only necessary to deal very slightly with the resting eye behind the spectacle lens. It is assumed that the axis of the eye corresponds with the axis of the spectacle lens, and with this arrangement, by means of the trial frame, one determines the highest visual acuity which can be obtained with an appropriate trial lens.

The test types, which must be at a sufficient distance (at least 5 m.), will be reproduced with sufficient accuracy at the posterior focal point F′ of the spectacle lens, which for the present will be considered to be a thin lens, and the refractive power of which, measured in diopters, will be $R_1 = 1/f_1$. If, then, this spectacle lens be placed before an eye with a refractive power R_{11} so that, according to fig. 16, a distance

$$\delta = S\,\mathbf{H}$$

exists between S the posterior principal point or vertex of the thin spectacle lens and the anterior principal point **H** of the eye, then it will follow that the refractive power R′ of the spectacled eye is

$$R' = R_1 + R_{11} - \delta R_1\ R_{11}$$
$$= R_{11} + R_1\ \{1 - \delta\ R_{11},\}$$

and it will be easily seen that

$$R' = R_{11},$$

or, that the refractive power of the bespectacled eye equals that of the normal eye, when

$$1 - \delta R_{11} = 0,$$

or when

$$\delta = \frac{1}{R_{11}} = f = 17.06 \text{ mm}.$$

The Distance from the Eye of Thin Spectacle Lenses.

If, then, the distance δ between the posterior principal point or vertex S of the thin lens and the anterior principal point **H** of the eye is equal to the anterior focal length f of the eye, or if, in other words, the anterior focal point **F** coincides with the vertex S of the thin spectacle lens, then the refractive power of the spectacled eye will equal the refractive power of a normal eye, and therefore the size of the retinal image of distant objects will be the same in both cases.

If δ be not the same as the anterior focal length f of the eye, then the size of the image

$$\beta = \frac{w}{R'}$$

upon the retina of the corrected eye will not be equal to the size of the image in the normal eye. But with only slight variations of δ the difference in the size of the retinal images will not vary very much from the normal.

Correcting Lenses.

In general, a lens which enables an ametropic eye to see distant objects clearly or, in other words, which reproduces them at the far point of the ametropic eye, is termed a correcting lens, and its refractive power equals the correction value D_1 of the ametropia.

Referred to the axial ametropia A of the eye and the distance δ the following relative values of the correction value R_1 can be deduced without any particular difficulty.

It follows from the condition that the focal point F′ of the spectacle lens shall coincide with the far point R of the eye, as will be seen from fig. 16.

$$H\,S + S\,F' = H\,\mathbf{R}$$
$$S\,F' = S\,H + H\,\mathbf{R}$$
$$f_1 = \delta + a$$

$$R_1 = \frac{1}{\delta + a} = \frac{\frac{1}{a}}{\frac{\delta}{a} + 1} = \frac{A}{1 + \delta A}$$

If now, for reasons which will appear later, we assign for δ a constant value which shall only differ slightly from 17.06 mm.,

$$\delta = 13.3 \text{ mm.} = 0.0133 \text{ m.},$$

then the following table of differences between the axial refractive error A of the eye and its correction value R_1 (both in diopters) can be set up.

A	R_1	A	R_1
+10	+8.83	− 6	− 6.52
+ 8	+7.23	− 8	− 8.95
+ 6	+5.56	−10	−11.54
+ 4	+3.80	−12	−14.28
+ 2	+1.95	−14	−17.20
− 2	−2.05	−16	−20.32
− 4	−4.23	−18	−23.67
		−20	−27.25

It may be pointed out here that not only is the refractive power of a lens measured in diopters, but also the abnormal refraction which is corrected by such a lens. We shall therefore have to deal with converging lenses, +1, +2, +3, . . . D., and with diverging lenses −1, −2, −3, . . . D. This system of numbering spectacle lenses is now universal (international), but it was only introduced in the seventies of last century (after the term "dioptrie" had been formed) by the Strassburg ophthalmologist, F. Monoyer. Before this the system of numbering was quite arbitrary. It was the focal length of the lens assumed to be thin, measured in the respective inches of the various countries. This is now quite antiquated, and it will suffice to take as an example the English inch,

$$1 \text{ inch} = 25.4 \text{ mm}.$$

The conversion of the number M of one system into the other will be obtained by

$$M_D = \frac{41.6}{M_{inch}}$$

if the refractive index of the glass for the thin lens be taken as

$$\mu^D = 1.528.$$

If the numerator be taken as roughly 40, then the numbers of one system have to be divided into 40,

in order to obtain with sufficient accuracy the corresponding number of the other system. Therefore the modern -8 D will equal the old -5 and the old number $+13$ will equal $+3$ D of the new system.

If we determine by means of a correcting glass the size of the letters of the test type which can still be recognised, we have obtained the absolute visual acuity S. This is determined by employing for the distant test type the strongest convex or the weakest concave lens with which the ametropic eye will obtain its maximum visual acuity.

Presbyopic Glasses.

In old-sighted (presbyopic) eyes we have to deal with a definite near distance a_1 between the object and the surface of the lens, and a distance between the image and the surface of the lens determined by the condition of the eye. We therefore obtain the refractive power R_p of the presbyopic glass from the fundamental formula

$$B_1 = A_1 + R_p$$

therefore

$$R_p = B_1 - A_1.$$

The value R_p measured in diopters is generally positive. It can only be negative when the image is nearer the lens than the object, and that occurs in the higher degrees of myopia. If we are dealing with an emmetrope who has become old-sighted (presbyopic), where, in the course of time, the distance b_1 of the near point has become great, and B_1 therefore small, then approximately

$$R_p = - A_1,$$

and we obtain

$$R_p = 3 \text{ D},$$

if we assume

$$- a_1 = 0.33 \text{ m.},$$

as the most convenient reading distance.

In general, the surgeon will have to ensure that the accommodation is used with reading (presbyopic) glasses, if only in order that the accommodation may be relaxed when regarding the more distant lateral parts of the reading or writing surface.

The Loupe Spectacle.

Finally, it is conceivable that the image formed by the spectacle may lie at infinity, so that parallel rays will enter the eye. The spectacle will then be employed in an exactly similar manner to optical instruments in the narrower sense of the term, and more particularly like loupes. For this reason these spectacles are termed Loupe spectacles. Their magnification V_l can be deduced from the relationship of their focal length f_l to the conventional value for the distance of clear vision 25 cm.

$$V_l = \frac{25 \text{ cm.}}{f_l} = \frac{1}{4} R_l$$

Spectacle Lenses of Finite Thickness.

It is only possible approximately to fulfil the assumption of thin spectacle lenses in the case of diverging lenses; even weak converging lenses require for a definite diameter of glass a certain average thickness d. If the two refracting surfaces are separated from each other by a distance d of definite value (not infinitely small), then the distance

$$s' = \text{S F}'$$

of the posterior focal point from the back surface of the lens is, in general, no longer equal to the focal length of the lens. This distance s' is of the greatest importance for the selection of the correcting lens, for, in such a case as well, the posterior focal point F′ of the spectacle lens must coincide with the far

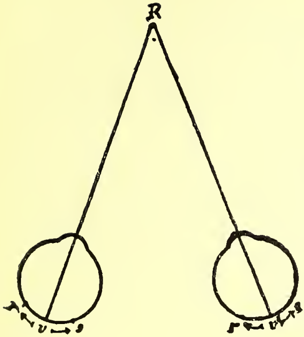


FIG. 15. The use of both eyes. $v\ R\ v'$ Angle of convergence for the object point R. $v\ v'$ Foveal depression in the left and right eye. $r\ r'$; $s\ s'$ corresponding points. Since $v\ r = v'\ r'$; $v\ s = v'\ s'$.

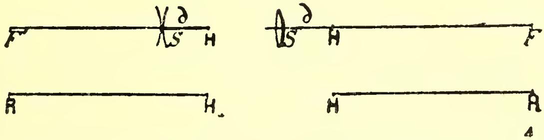


FIG. 16. The connection between the axial refraction A, and the value of the correction R, of the ametropia.

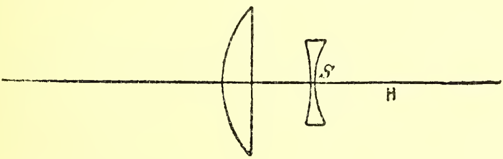


FIG. 17. The arrangement of the components in a telescopic spectacle.

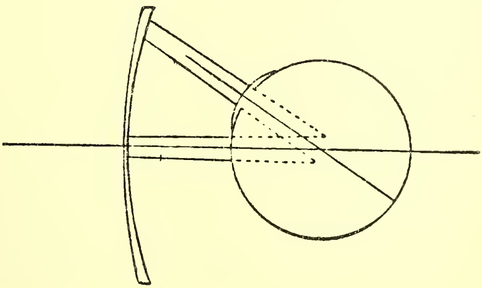


FIG. 18. The bundle of the visual lines behind the spectacle lens.

point **R** of the eye being corrected. Thus one obtains again as before, on page 35,

$$s' = \delta + a,$$

where δ is similarly measured from the back surface of the lens.

Vertex Refraction.

The above mentioned relationship renders it desirable to introduce for $\frac{1}{s'}$, the new term vertex refraction,

$$\frac{1}{s'} = A_s$$

where the index s stands for the distance of the object, so that in the above case we have to deal with A_∞. We obtain, then, as against thin lenses the somewhat altered relation

$$A_\infty = \frac{A}{1+\delta A}.$$

The refractive power of such a lens of finite thickness is different from the vertex refraction. It would not be difficult to determine these differences—but it is unnecessary for our purpose. Only this may be mentioned, that the refractive power of a spectacle lens of finite thickness determines the size of the image upon the retina of the wearer.

It will be easily understood that, if only for reasons of weight, the average thickness of simple converging lenses must not exceed a certain amount, and therefore the omission of the difference between image distance s' and focal distance f_1, or between lens surface refraction A_∞ and refractive power R_1 in such lenses will be inexact, but in most cases the error will not be very great. In simple, thin, diverging lenses the difference is exceedingly small.

The Telescopic Spectacle.

The matter, however, is quite different when we come to deal with spectacles constructed of a pair of lenses. Such spectacles have been introduced recently, and on account of their superficial resemblance to a Galilean telescope, have been termed telescopic spectacles. In these, the field of vision of usual size has been sacrificed in favour of an enlarged retinal image. For this purpose two lenses of different signs and separated by a finite (not infinitely small) distance, have been taken, the converging lens is directed towards the object, while the diverging lens is next the eye. Since the refractive power of the individual lenses (R_1 and R_2) are comparatively great, the lens surface refraction A_∞ is markedly different from the refractive power R_{12}, of the whole telescopic spectacle, and it is not possible to consider these two values approximately equal. In any case an enlargement of the retinal image is obtained which may assume very high values (180% and more).

This form of spectacle was originally introduced —towards the end of the 18th century—for highly myopic eyes. But it can be very usefully employed in ametropias of lower degree. It can be used as a correcting or as a presbyopic glass, and, finally, as a loupe spectacle having a specially long object distance. This subject will be dealt with more fully later on (pp. 83-90).

Other forms of double lenses have been constructed, but as they have not been found practical it is needless to discuss them here.

Spectacle Lenses for the Moving Eye.

However important the above considerations were for the investigation of the various ametropias, they do not suffice for ordinary practice. For this

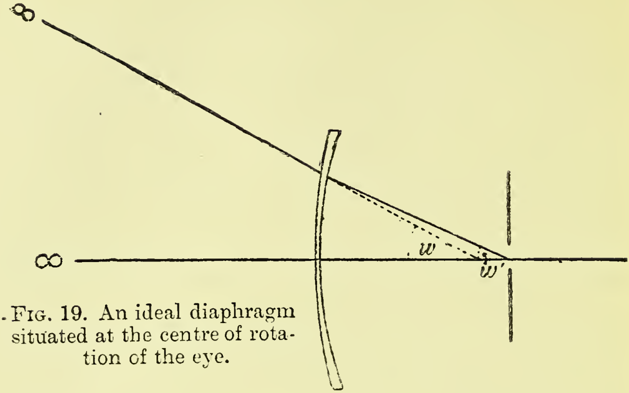


FIG. 19. An ideal diaphragm situated at the centre of rotation of the eye.

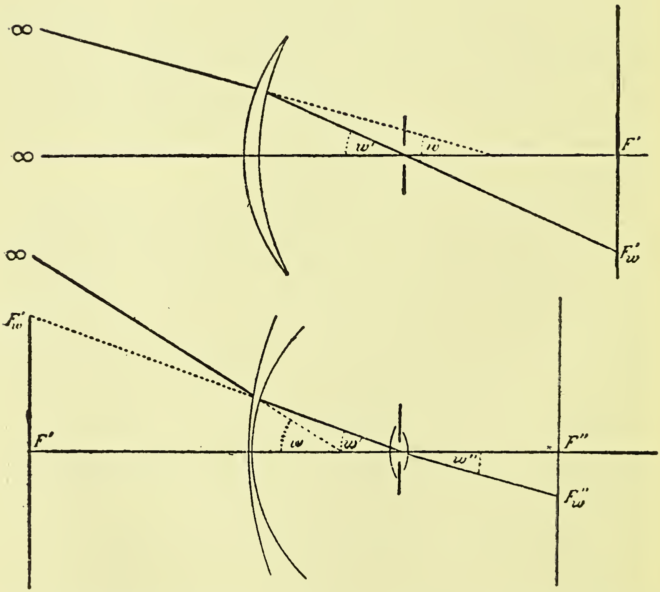


FIG. 20. The determination of the course of the principal rays on the image side; the case of a converging lens above, and below of a diverging lens.

purpose the spectacle and the moving eye must be considered together, for, as has been stated previously, in ordinary vision the eye is directed towards the object which has attracted the attention of the observer.

We have to assume that the eye behind the spectacle moves about its centre of rotation, and that its visual line is brought into the most varied positions, while the spectacle lens maintains an unaltered position with regard to this centre of rotation. In fig. 18 this is represented in the case of a diverging lens. It will be seen from fig. 19 that the problem depends for its answer upon the question: What would happen if we introduce a diaphragm with a small aperture behind an optical system, and allow rays of great inclination ($2w' \leqq 70°$) to enter?

CENTERED OPTICAL SYSTEMS WITH SMALL APERTURE.

The Alteration of the Direction of the Principal Rays.

The answer to the above question resolves itself into two parts according as to whether the alteration in direction of the principal rays is considered, or the image formed along these rays. Firstly let us enquire into the direction of the corresponding rays on the side of the object and of the image respectively. The general statement can be made that in every optical system, used as a spectacle, the direction of each principal ray on the image side corresponds to one on the object side, but which, in general, differs from it. If the centre of rotation of the eye lies on the axis of the lens, that is if the spectacle be properly centred, as of course it ought to be, then there will be one specially favourable direction (namely, along the axis), along which the

directions of the rays on the object and image side will coincide with each other. All other directions of rays on the eye side, *i.e.* those which, with the axis, make a finite (not infinitely small) angle w' – differ, generally, and always in thin spectacle lenses, from the direction of the rays on the object side, with the angle w between the axis and the ray; and they are arranged symmetrically about the favoured axial direction. The difference $w' - w$ is frequently termed the prismatic effect of the peripheral portion of the lens, and its presence is looked upon as a defect of the outer parts of the glass. This view is wrong, because the wearer of the glass cannot immediately become conscious of the direction of the rays on the object side; he is only aware of the rays on the image side, and is therefore incapable of comparing the two directions, and thus of rendering manifest to himself the alteration in direction of the lateral parts of the field of vision. In all compound instruments such as the microscope and telescope it is possible to obtain equally good information from the lateral parts of the field as from the central portion, provided the definition of the image is not much reduced. It is even possible to say that the power of magnification which these instruments possess is directly due to the prismatic effect in the lateral portions of the image. In spectacle lenses the angle of altered direction is, generally, only moderate, and is either not noticed at all by the wearer, or only for the first few days. But it is of the utmost importance that there shall be for the direction of each principal ray on the object side only one principal ray direction on the image side, and *vice versâ*. It is possible, by means of photography, to determine exactly, even in complicated cases, the course of the principal rays after passing through a lens.

It will be clear from fig. 20 that in order to obtain the point of transmission F'_w of an inclined ray w at

the posterior focal plane of a converging lens, all that it is necessary to do is to place a photographic plate at right angles to the principal axis of the lens at F′. It is impossible to do this in the case of a diverging lens, because the posterior focal plane is virtual. But this difficulty can be overcome by a method first proposed by C. A. Hegner. It consists in using a photographic objective free from distortion, and having a flat field. The entrance pupil of this objective is placed behind the lens at a point corresponding in position to the centre of rotation of the eye, and in this position the virtual points of transmission F'_w in the posterior focal plane will be the object photographed upon the plate at F''_w. It will then be possible to measure the distance F'' F''_w, which is what we require, as the reduction due to the photographic objective may be considered as known.

Images produced along Oblique Principal Rays.

Having dealt with the most important points relating to the alteration of direction of the principal rays, it is now necessary to speak of the quality of the image produced along these principal rays. If one take, as in fig. 21, an object point situated on the thicker principal ray, and from this point two other rays be taken only slightly inclined to the principal ray (corresponding to the small opening of the assumed diaphragm at the centre of rotation of the eye), it is not at all certain in the general problem whether the two corresponding directions in the eye space will have a point of intersection, as they might possibly cross each other.

As a matter of fact, this may be the case if the two slightly inclined rays be chosen quite arbitrarily. Malus' theory states that an optical system can only so alter entering spherical waves that there will be corresponding continuous wave surfaces;

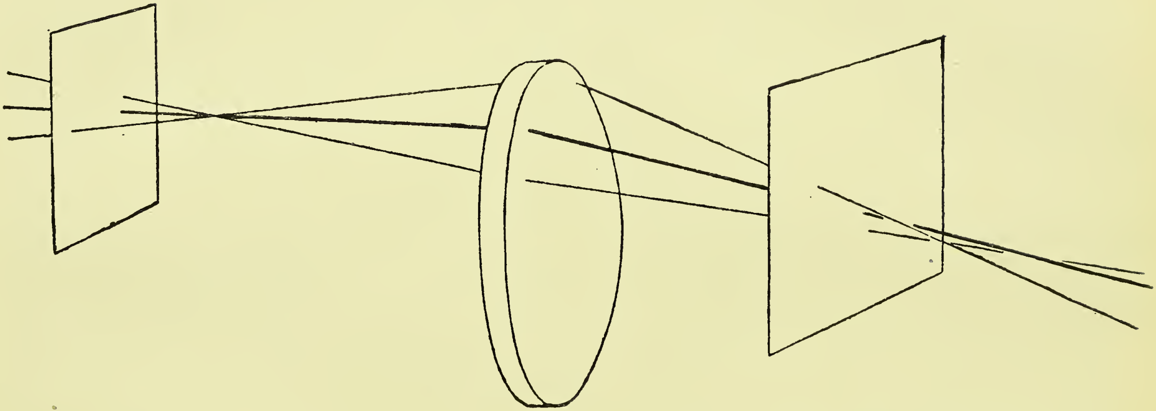


Fig. 21. The astigmatism of oblique rays. Any two rays near the thickly drawn principal ray do not intersect this ray after refraction but cross it.

and from this the following important conclusion can be drawn, which is illustrated in fig. 22, namely:—There must be, on the object side, two plane beams at right angles to each other, each containing the principal ray, corresponding to which there will be two plane pencils, at right angles to each other, on the image side, and there will thus be in each one point of intersection. These points, which lie at a finite distance from each other upon the principal ray, are termed *Focal Points*, the two planes at right angles to each other *Principal Planes*, the bright lines arising in them *Focal Lines*, and the whole appearance of the pencil due to the oblique direction of the rays is known as the astigmatic deformation of an originally homocentric pencil—or, shortly, as the astigmatism of oblique pencils. It is very extraordinary that this is the most common condition even when any centrally used optical system is employed. How, then, can the principal planes be determined?

In the problem as applied to spectacles, one can assume that they are properly fitted, that is, well centred, and that the centre of rotation will lie upon the axis of the system. Further, the problem may be limited to axially symmetrical centred systems, as used for the correction of axially symmetrical ametropic eyes. Taking these assumptions for granted, then any principal ray will run in a plane containing the axis of the system, a meridional plane. It is only necessary to deal with one such meridional plane, and to investigate what takes place in it in order to know them all; for by rotation upon the axis, which is, of course, permissible in an axially symmetrical system, the selected meridional plane will correspond to any other. By thus simplifying the problem it becomes easy to determine the position of one principal plane. If we allow a slightly inclined ray to proceed from the object point

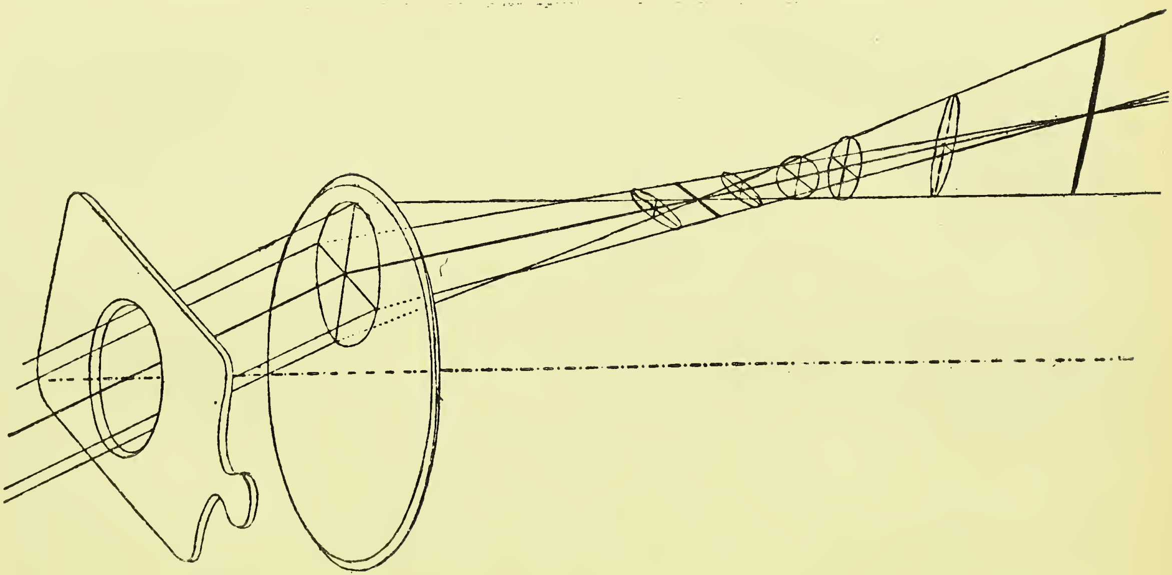
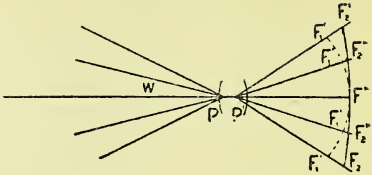


FIG. 22. The astigmatism of oblique pencils. The astigmatic deformation of an originally homocentric pencil caused by passing obliquely through a lens.

in the meridional plane, then according to the law of refraction* it cannot leave this plane, and therefore after emerging from the system it must intersect the principal ray. Therefore the first principal plane will coincide with the meridional plane, and thus it becomes easy to determine the second principal plane, for it must pass through the first one at right angles along the principal ray. It has become customary to speak of the plane beams of the meridional plane as tangential (t), and those of the second principal plane as sagittal ($ſ$). What, then, will be the consequence of the occurrence of focal lines which cross each other at right angles? They will cause peculiar diffusion figures, which must now be investigated. The finite distance between the two focal lines depends upon the inclination w, w' of the principal ray; along the axis it is zero, for naturally there can be no astigmatism of oblique rays, there must be the formation of a true image; but with increasing obliquity w, w', the distance between these two focal lines increases. If the position of the focal lines in the meridional plane of a series of principal rays of varying obliquity be found, and then the points of the t-bundles be joined together, and also those of the $ſ$-bundles, by means of curved lines, then one will obtain, as in fig. 23, two curves, the image curves of the t- and $ſ$-bundles of this meridional plane, and these, as is seen, will touch each other upon the axis. If this meridional plane be now rotated about the axis, which is permissible in axially symmetrical systems, we will obtain the image surface of the t- and the $ſ$-bundles, *i.e.* the geometrical positions for those points at which anything like an image of infinitely distant objects will be formed, namely the positions of the focal lines

* In refraction, the incident ray, the normal at the point of incidence and the refracted ray must lie in the same plane.

FIG. 23. The astigmatic image surfaces of an uncorrected system. The dotted curve represents the section through the surface of the *t* pencils, the continuous line that through the surface of the ∫ pencils.



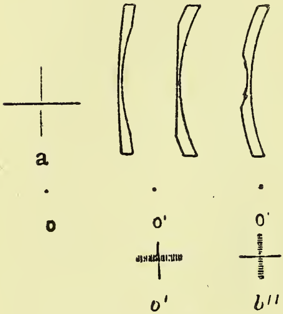


FIG. 24. The appearance of astigmatism. *a* the object cross, *b'* the image cross when the ground glass is in the surface of the Sagittal or ∫ pencil, *b''* when in the surface of the tangential or *t* pencils.

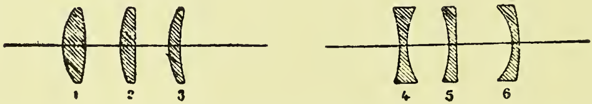


FIG. 25. 1, 2, 3, converging lenses, centre thicker than the edge. 4, 5, 6, diverging lenses, centre thinner than the edge. 1 biconvex, 2 plano-convex, 4 biconcave, 5 plano-concave, 3 convex, 6 concave meniscus (little moon).

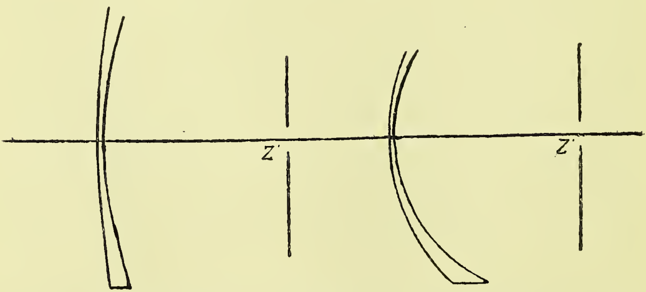


FIG. 26. The Ostwalt and Wollaston forms of a lens of −5 D.

which for the *t*-bundles will run peripherally, and for the ∫-bundles radially. It will easily be seen that this must be so. The focal lines of the *t*-bundles of the selected meridional section were perpendicular to the plane of this section, and therefore coincided with the direction of the tangents of a circle which would be produced by revolving the focal point of the bundle around the axis. The focal lines of the ∫-bundles were in the plane of the selected meridian itself, and therefore had a direction towards the axis of rotation. In rotating the meridional plane the focal lines will assume all the radial positions in a circle formed by the path of the focal point of the ∫-bundle.

If, then, an attempt be made to obtain an image of an object, say any straight line, by placing a ground glass as close as possible to the image plane (curve), what will generally be seen is a blurred image, both in the plane of the *t*- and of the ∫-pencils, for every point of the straight line is drawn out into a line which runs either peripherally or radially. It will be easily seen that there will be two positions of the line in which the focal lines of each system will coincide with each other, thus giving the appearance of a clear image. This will be for the plane of the *t*-pencils, the curves directed peripherally, that is, all the circles concentric to the axial point; and for the plane of the ∫-pencils the radially directed curves, or, in other words, the whole bundle of rays drawn in the plane of the object, from the axial point. These object lines are to be regarded, according to A. Gullstrand, as reproducible; they do not correspond point for point with their reproduction upon the ground glass in the image plane; but, owing to the deficient union of rays in the astigmatically deformed bundle, only line for line, although this is of considerable use. The point of intersection of two reproducible lines

and its immediate neighbourhood, here a cross, can never be exactly reproduced, as seen in fig. 24, by a system uncorrected for the astigmatism of oblique rays, since one of the lines will always be blurred when the other appears clear.

This astigmatism of oblique bundles occurs, in general, in all systems employed with oblique rays, and naturally also in spectacle lenses. The first attempt made to correct it occurred in 1840, when J. Petzval, of Vienna, calculated his photographic portrait objective so as to be free from this error, but eleven years before this H. Coddington, of Cambridge, had published his extensive theoretical researches upon the nature of the image formed by oblique pencils.

The Problem for the Spectacle Lens.

The two problems which, as we have already seen on page 22, arise in dealing with ordinary vision, namely, the question of the clearness of perception, and the direction of the object perceived, arise also when seeing through instruments. And we shall now have to discuss how far vision, in an ametropic eye, can be improved by a fixed system, and how the direction of the object perceived appears to be altered. This last question has already been indicated on page 43. To begin with, it must be stated that one of these problems must be preferred to the other, for both cannot be perfectly solved by a comparatively simple system, such as one built up of two spherical surfaces. Under these circumstances the chief weight should be given to the clearness of perception, for the first requirement must be to see clearly, and only secondarily is it necessary to investigate how perception, under these conditions, differs from the perception of the emmetropic observer in the same place.

With the above we conclude the discussion of the alteration of direction and the formation of the image by means of oblique rays through an optical system of ordinary construction. It only remains to add that, in applying these conditions to the case of the spectacle lens, this latter, in ordinary use, is employed almost exclusively with oblique rays. As soon as one looks at an object of finite extent with the head held steady the eye must turn upon its centre of rotation and the peripheral parts of the glass must be used. This always was so, however unconscious of it the wearer of glasses may have been.

Attempts to Correct the Astigmatism of Oblique Rays in Simple Spectacle Lenses.

This astigmatism of oblique pencils, and the loss of clearness caused by it, was noticed long ago by careful observers, as, for instance, by the English physician, W. H. Wollaston, about the year 1804, at a time, therefore, when both the cause and the form of this error of optical instruments were quite unknown. He suggested spectacle lenses of better form, which he called periscopic (looking about) glasses, but they were only adopted very slowly. Even much later efforts to improve the quality of the image of oblique rays through spectacle lenses, whether merely experimental attempts or of scientific design, met with only very slight adoption. And this is very curious. The explanation may be that the users of glasses did not realise how much technical methods might improve the function of the lateral portions of their glasses, but probably the real reason was that the very inexact method of determining the power of spectacle lenses scarcely afforded the wearer the highest efficiency in the central parts of the field, let alone any attempt to

obtain perfect images from the peripheral parts. And this would not have been very easy, for a good observer would have had to decipher his test type by means of oblique rays, and it would have been quite impossible to make such an exact determination without the help of not very simple mechanical appliances.

Such improvements as were attempted were in the direction of being able to supply, in addition to the old equal-sided and plano-spherical glasses, bent or meniscus-shaped glasses.

At first these were of the periscopic type, and were generally so arranged* that the convex surface in diverging lenses and the concave surface in converging lenses were of a fixed radius of 40 cm. Later, for medium strengths, other meniscus forms were introduced, having fixed radii of $17\frac{1}{2}$, $11\frac{1}{2}$, 9 and $4\frac{1}{2}$ cm. Glasses of the latter curves were termed half-Coquille and Coquille† glasses.

The reason why, in addition to these lenses, which mostly gave much better peripheral images, the older equal-sided and plano-spherical glasses were also supplied, was because the public desired to obtain the cheapest possible glasses. Unfortunately, for a long time, only the cheapest glasses were used to help the eyes, and consequently theoretical possibilities and mechanical improvements were much neglected.

Leaving aside the historical evolution, for one finds no gradual improvement in the glasses generally used, we will deal here only with the theoretical possibilities.

The requirements necessary may be expressed as follows. It is necessary for a given lateral

* This arrangement was not proposed by Wollaston but apparently became a sort of tradition among opticians.

† I have been able to trace this expression back to about the middle of the sixties.

deviation of the eye, say $w' = 30$ to 35 degrees, to remove from a spectacle lens of given refractive power the astigmatism of oblique pencils. The only method which can be employed for this purpose is in the choice of the shape of the lens; the correct amount of bending. If the inner surface be determined by its refractive power R″, then, from the given refractive power R_1 of the whole lens (assumed to be a thin lens)

$$R_1 = R' + R''$$

and the anterior surface by the formula

$$R' = R_1 - R''.$$

We must investigate whether this means suffices to remove the astigmatism of the oblique pencils for the given inclination of the principal ray.

The Determination of the Position of the Centre of Rotation of the Eye.

Before the above question can be answered, we must determine the position of the centre of rotation of the eye. From previous explanations the ideal requirement would be that the retinal image of the spectacled eye should be of exactly the same size as in an emmetropic eye. For this it is necessary that

$$\delta = S\,\mathbf{H} = 17.06 \text{ mm.}$$

If the distance from the anterior principal point of the eye H to the corneal vertex S be considered, then

$$S\,\mathbf{S} = 15.71 \text{ mm.}$$

Taking into consideration that the centre of rotation Z′ is 13 mm. behind the surface of the cornea and on the axis, we shall obtain for the distance from the posterior surface of the lens

$$S\,Z' = 28\tfrac{3}{4} \text{ mm.}$$

As was already mentioned on page 33, the alteration in the size of the retinal image is only very slight, with small variations in the ideal value of δ. It will therefore be possible to lower slightly the value δ, and with this the distance S Z′, without any serious change in the size of the retinal image. The limit for this diminution is such a distance between the inner surface of the glass and the eye as will insure the eyelashes not touching the lens. If we take for this distance, *cf.* fig. 18, page 39,

$$S\ \mathbf{S} = 12 \text{ mm.},$$

we obtain a total distance

$$S\ Z' = 25 \text{ mm.},$$

and this is a fixed quantity which can be verified between the centre of rotation of the eye and the inner surface of spectacle lens. The diminution of the original value has two advantages: one, that a simple lens system of given focal length can be more easily corrected for the astigmatism of oblique pencils if a shorter diaphragm distance can be chosen. And, secondly, a lens of smaller diameter will suffice for the same angle of rotation, and therefore the diminished value of S Z′ will give us smaller and lighter lenses. It was stated on page 43 that the centre of rotation of the eye Z′ is assumed to lie on the axis of the lens; this is termed the primary position of the centre of rotation of the eye, and one speaks of a centrally used spectacle lens.

The two Forms of Exactly Reproducing Spherical Spectacle Lenses.

Having thus determined the distance from the centre of rotation of the eye Z′ to the spectacle lens, it is possible to define the question more exactly, whether, under these circumstances, it be possible to correct the astigmatism of oblique pencils in a

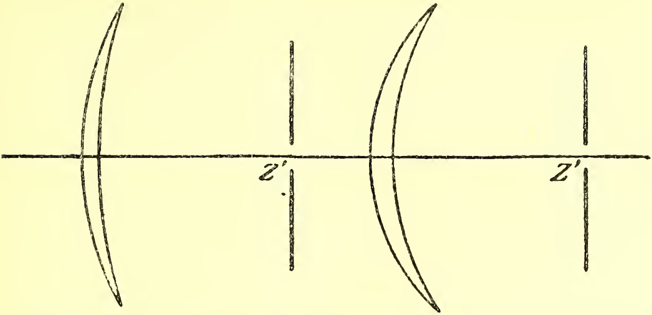


FIG. 27. The Ostwalt and Wollaston forms of a lens of +5 D.

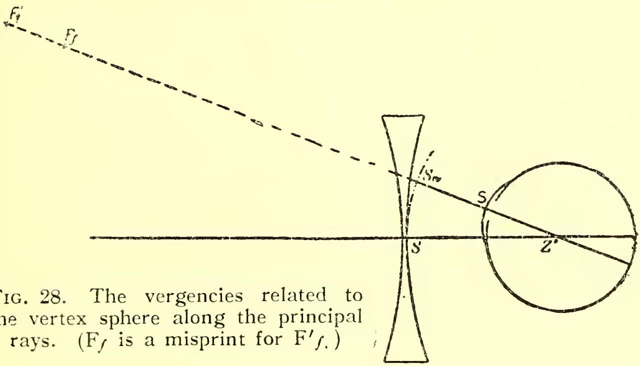


FIG. 28. The vergencies related to the vertex sphere along the principal rays. (F_f is a misprint for F'_f.)

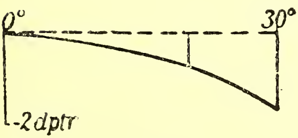


FIG. 29.

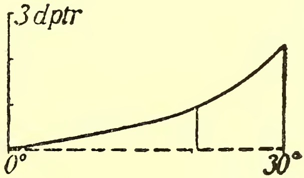


FIG. 30.

The astigmatism of oblique pencils of an equal-sided dispersing (−5 D) [Fig. 29] / collecting (+5 D) [Fig. 30]
Spectacle lens (——) and of a corresponding dispersing (−5 D) / collecting (+5 D)
Spectacle lens (— — —) of Ostwalt form.

spectacle lens of given power, and for any finite angle of vision w'. The answer is: Yes, it is possible to obtain thin spherical lenses of certain forms in which, within certain definite limits of refractive power, the astigmatism of oblique pencils for finite angles w' is absent, when the centre of rotation of the eye is 25 mm. behind the posterior surface of the lens. Such glasses may be termed exactly reproducing (lenses of punctiform images), in order to distinguish them from ordinary glasses, in which exact reproduction is limited to the paraxial region, while in all vision directed obliquely there is an astigmatic distortion of the small pencils.

There are two forms of exactly reproducing glasses for every refractive power, and these are distinguished from each other by the extent of the bending. The less bent form may be called the exactly reproducing lens after Ostwalt, the more strongly bent ones after Wollaston. The accompanying figures (26 and 27) will illustrate the appearance of these two forms of lenses having a refractive power respectively of -5 D and $+5$ D. It is impossible here to go fully into the question of the relationship between the form and the refractive power of exactly reproducing lenses; but it may be stated that there is no single outer radius of constant length which will give an exactly reproducing lens for all refractive powers used in spectacle lenses; in other words, the radius of the outer curve will vary with alterations in the refractive power.

In order to give some idea of the extent of the astigmatism of oblique bundles in glasses as ordinarily worn, and the extent to which this error can be avoided, we will give, as an example, the mathematical quantity of this error and a graphic representation of these results.

The astigmatism of oblique pencils will be the difference in convergence value measured in the t-

and in the f-sections. In order to obtain exactly comparable results, one will always reckon the distance, *cf.* fig. 28, to the two focal points F'_t and F'_f from a point S_w, which will lie upon the selected principal ray at a distance of 25 mm. from the centre of rotation of the eye Z'; or, in other words, one can refer the astigmatism of oblique pencils to the sphere which is drawn through the inner vertex of the lens S with Z' as its centre. The astigmatism will then be obtained as the difference between the two convergence values expressed in diopters; and these are data which can be easily applied by the oculist.

In figs. 29 and 30 are examples of the astigmatism of oblique pencils, as it will occur in equal-sided bi-convex and bi-concave spectacle lenses placed at a distance of 25 mm. from the centre of rotation. At the same time, the extremely small astigmatism produced by an exactly reproducing lens of Ostwalt form has been marked on these same figures. In both cases an extreme rotation of the eye of only 30° has been taken.

In order to realise this marked difference, one must remember that the astigmatism of the eye is corrected as soon as it exceeds 0.25 D or 0.5 D. Astigmatism of oblique pencils will therefore be very noticeable in the peripheral field of vision of equal-sided (bi-convex and bi-concave) spectacle lenses.

The significance of the removal of this astigmatism of oblique pencils will be rendered quite obvious if we take photographs in monochromatic green light of test types under conditions exactly similar to those which occur when objects are viewed obliquely through an ordinary spectacle lens. The photographs should be taken with varying degrees of obliquity in order to show the increasing influence of this error on the image.

Such photographs are reproduced in the columns *a* and *b* of the topmost section of the plate at the end of this volume. Under *a* are photographs taken through a bi-convex lens of +5 D, and under *b* through an exactly reproducing lens of Ostwalt form, of equal power. The inclinations w' of the principal rays on the image side are, in both cases, 0°, 10°, 20°, 30°, and it is obvious that these two lenses produce similar images only along the axis. With an inclination of only 10°, and still more with stronger inclinations of the principal rays, the image formed by the bi-convex lens becomes blurred, and in such fashion that fine types become unrecognisable before coarse ones. With exactly reproducing lenses under the conditions of the experiment the image formed by the peripheral parts of the lens is quite as useful as that from the centre.

As far as the relationship between the two forms, those of Ostwalt and of Wollaston, is concerned, they may be considered for practical purposes to be equal in effect in dealing with oblique principal rays. Once the astigmatism of oblique bundles is corrected for a definite rotation of the eye ($w'=30°$), then, as will be seen in figs. 29 and 30, for the Ostwalt form of ±5 D, it will be corrected for all lesser inclinations of principal rays. This peculiarity, which may be termed the absence of zones, in the correction of astigmatism, is, for all practical purposes, common to axially symmetrical lenses of both systems to the same extent. With errors of distortion we shall have to deal later, but it may be stated here that, in this respect, there is a difference between these two forms, but that it is of no great importance.

There remains one further point to be dealt with in connection with the above answer, namely, the limits of refractive power within which an exactly

reproducing lens is possible. Taking a fixed refractive index at, say,

$$\mu_D = 1.52,$$

then it will only be possible to correct the astigmatism of oblique pencils if the refractive power R_1 of the lens be within the limits of

$$-25\ \mathrm{D} \leqq \times R_1 \leqq 7\tfrac{1}{2}\ \mathrm{D}.$$

It will be apparent without further explanation that this limit will suffice for the negative side, and also for most cases of congenital hypermetropia. But, on the other hand, it will not suffice for most cases of cataract glasses. All ordinary cataract glasses, having spherical surfaces and being worn at such a distance that the eyelashes do not touch the lenses, must consequently have astigmatism of oblique pencils. We shall have to deal later with the methods employed to produce cataract glasses having exact reproduction in a field of vision of finite extent.

The Image Surface of an Exactly Reproducing Spherical Spectacle Lens.

If, by the means above indicated, we have succeeded in removing the astigmatism of oblique pencils in exactly reproducing spectacle lenses, there still remains the interesting question of the shape of the surface upon which such a lens will form an image of an infinitely distant plane. In this connection one can apply a general law on the curvature of image surfaces near their apices, which was formulated by the mathematicians previously mentioned, H. Coddington (1829), and J. Petzval (1843), and is known as Coddington-Petzval's law.

When the astigmatism of oblique pencils is corrected in a centred system composed of lenses

L_1, L_2 . . . . . . the image will lie on a definite curved surface; and in proximity to the axis, the curvature of $\frac{1}{R}$ of this surface is entirely independent of the thickness of the lenses and of the distance between them as well as of the position of the diaphragm, the only factors which do influence it are the refractive indices μ_1 μ_2 . . . . . . and the focal lengths f_1 f_2 . . . . . . in this manner

$$-\frac{1}{R} = \frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} + \dots\dots$$

It is impossible to determine generally how far this formula, deduced from infinitely small inclinations w, will apply for finite values, but as the result of many experiments with simple spectacle lenses it has been found that this Coddington-Petzval law can be applied also to finite inclinations, and therefore the image surface of a centrally used exactly reproducing spectacle lens is to be regarded as a centred spherical surface, the radius of which can be deduced from the formula,

$$-\frac{1}{R} = \frac{1}{\mu f_1},$$

as

$$R = -\mu f_1,$$

or, put into words: In a centrally used exactly reproducing lens the image of a distant plane will lie upon a spherical surface passing through the posterior focal point F′, the radius of which can be obtained by multiplying the focal length by the refractive index of the glass.

Let us take an exactly reproducing glass as it would be used for the correction of an ametropia,

i.e. so centred before the eye that the focal point F′ of the glass coincides with the far point **R** of the eye. Under these conditions the eye will perceive objects just as clearly laterally as it would centrally, if the image surface of the lens coincided with the far point sphere of the eye. This, however, is not the case with lenses of medium refractive power, because the curve of their images is flatter than the far point sphere of the eye which is being corrected. The difference between these two surfaces is, however, slight, and is further concealed by the depth of the image, unless the object looked at be very fine. It is possible by carefully selecting the correcting lens to make it necessary to use only a small effort of accommodation (less than .5 D), in order to render both the central and peripheral parts of the field of vision equally clear. It ought to be mentioned that in contradistinction to the true optical instruments (telescope, microscope, photographic lens), the curvature of the image in correcting spectacle lenses is, in general, too small.

Exactly Reproducing Presbyopic Glasses.

An exactly reproducing correcting lens does not theoretically retain its quality of exact focussing if the object point be assumed to lie at a finite distance on the principal ray of the object side. On the contrary, astigmatism of oblique pencils would occur along the ray. The reason that it should not be more obvious, and does not require to be considered in lenses of medium power, is because, firstly, an exactly reproducing correcting glass of medium strength does not show a great amount of this astigmatism when the object is brought nearer, secondly, the lack of accurate observation on the part of the wearer, and, thirdly, because most near objects looked at are not fine enough to make this error

noticeable. If we wish to produce a perfect presbyopic glass, we shall have to repeat the calculation for a given refractive power R_1 and a given axial distance *a* of the object, in order to find the proper amount of bending which would correct the astigmatism of oblique pencils under these altered circumstances. We would then find, for each refractive power, two different exactly reproducing lenses which might be distinguished from each other as the Ostwalt and the Wollaston form of presbyopic glasses. The latter would scarcely differ at all, the former somewhat more, from a correcting lens of equal sign and equal refractive power. For these exactly reproducing lenses closer investigation would also show that the curvature of the image field, for most of the usual strengths of glasses, is too weak. The limits within which these lenses are applicable can be determined in the same manner, and we find that the possibilities in spherical lenses of moderate thickness are sufficient to be able to supply exactly reproducing presbyopic glasses for the more frequently occurring degrees of ametropia. There is, however, no possibility of being able to produce strong near glasses for aphakic eyes with a satisfactory freedom from the astigmatism of oblique pencils, if we limit ourselves to single spherical lenses of moderate thickness. We shall therefore have to return to this subject.

Exactly Reproducing Loupe Glasses.

Loupe glasses are required chiefly for eyes whose near point is situated a great distance away. Compared with correcting lenses the relationship between the object and image distances are exactly reversed, and therefore it will cause no surprise to find that the limits are entirely different. Further, it is obvious that only converging lenses need be

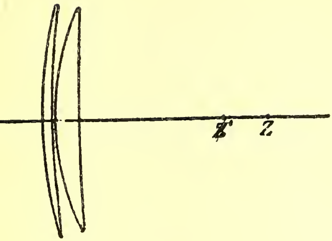


FIG. 31. An additional glass to a cataract lens.

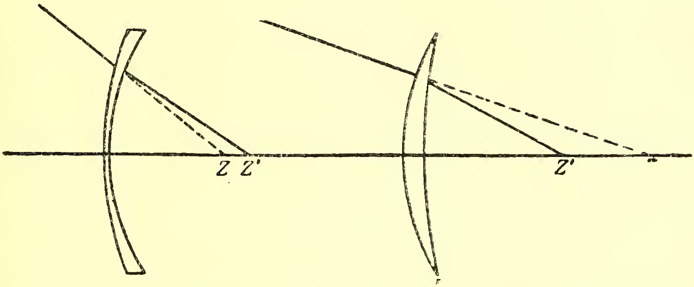


FIG. 32. The displacement of the centre of the visual direction by a
diverging converging
lens

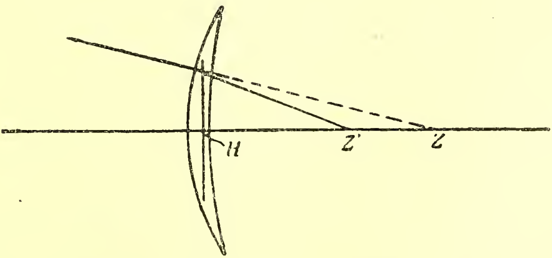


FIG. 33. The determination of the ratio of the tangents of an ideally thin lens.

considered. With ordinary glass we have a limit of about

$$R = +11 \text{ D}.$$

With reference to the curvature of the image field, the remarks previously made (p. 38), according to which the loupe glasses resemble optical instruments proper, will apply. They have to reproduce the focal plane at the infinitely distant plane, which is the near point surface of the eye requiring their help. This they cannot accomplish because their images are too much curved. There is no method of overcoming this curvature of the image in single spherical lenses.

Exactly Reproducing Additional Glasses.

For the use of presbyopes and those requiring loupe glasses, the employment of additional lenses to be placed in front of their ordinary correction is frequently very useful. This method has fallen into unmerited disuse. If a distance $-a$ for near work be assumed, then a lens having a refractive power of

$$R = A$$

will be able to reproduce the object surface at an infinitely distant plane, if the unavoidable curvature of the image surface be neglected. The distance a will generally lie between the limits of

$$0.20 \text{ m.} < a < 0.75 \text{ m.}$$

and therefore only weak converging lenses of

$$5 \text{ D} > R_{vv} > 1.33 \text{ D}$$

have to be considered, and, further, it is necessary to remember that in dealing with oblique pencils in such an additional lens the apparent centre of rotation, *i.e.* the centre of rotation reproduced by the correcting lens on the object side, is the diaphragm

position. It is generally possible to correct an additional lens for the astigmatism of oblique pencils, and thus to obtain an exactly reproducing additional glass. Such additional glasses are especially useful for placing in front of expensive correcting lenses, and also, when two different working distances are required, as, for instance, reading and piano playing. (Bifocal lenses are dealt with on page 92.)

The Relationship between the Directions of Vision on the Object Side and Eye Side (the Alteration of Perspective) for Spectacle Lenses of Medium Thickness.

To determine the alterations in the directions of vision caused by a lens of medium thickness, the following method may be employed. First, it is necessary to determine the position of the apparent centre of rotation Z on the object side of the glass. As will be seen from fig. 32, this is the point which corresponds to the image Z of the physical centre of rotation Z′ of the eye formed by the spectacle lens in the object space. It will be obvious that any ray which, in the eye space, will traverse the centre of rotation Z′ must be directed towards Z in the object space. If we have to deal with the object space of a spectacle lens, the centre of rotation of the eye will be replaced by its apparent position, and this replacement may be termed *the displacement of the centre of the directions of vision*. The next question that will arise will be : What alteration in the size of the angles will correspond to any alteration in the direction of the oblique rays, and to make this clear it will be well to consider first the effect of an ideal thin lens, and then transfer the results to the lenses of moderate thickness, as met with in practice.

The Alteration in Direction in Thin Lenses Free from Distortion.

In the ideal thin lens of fig. 33 it is assumed that the two principal planes fall into one plane, and that the angular alterations follow the same law as for the paraxial region. For purposes of illustration we will consider a converging lens, although exactly the same will hold for a diverging lens. In such a case the fundamental formula of page 6 will apply, under all conditions, namely

$$\frac{1}{a} = \frac{1}{b} - R_1 = \frac{1 - b\,R_1}{b}$$

in which

$$a = \text{H Z}; \; b = \text{H Z}'.$$

From the transposed formula

$$a = \frac{b}{1 - b\,R_1}$$

it will be seen that the apparent centre of rotation of the eye Z will be nearer the glass in a diverging lens ($R_1 < 0$) than the centre of rotation of the eye Z′ itself, while in converging lenses ($R_1 > 0$) it will be farther away from the lens. For any one height of transmission h of the common principal plane we obtain

$$\tan w' = \frac{h}{b}; \; \tan w = \frac{h}{a}$$

and therefore

$$\frac{\tan w'}{\tan w} = \frac{a}{b} = \frac{1}{1 - b\,R_1}$$

This expression, the ratio of the tangents, is therefore less than 1 for diverging lenses, and greater than 1 for converging lenses, or, in other

words, a short-sighted eye, through its glass will obtain from the same object smaller visual angles, and a far-sighted eye greater visual angles, than a normal-sighted one; if the apparent centres of rotation of the ametropes be at exactly the same distance from the object as the physical centre of rotation of the emmetrope.

For the assumed condition the law of angular variation

$$\frac{\tan w'}{\tan w} = \frac{1}{1 - b\,R_1} = \text{const.},$$

will also hold for angles of finite size, and therefore we can apply here a remark made when dealing with the eye alone. It will be remembered that one can term a perspective, the total of all the directions which can be drawn from a definite centre to all the points of the object. In the three cases described above the perspective on the object side is identical, or, in other words, the object will appear under the same angle from the apparent centres of the myope and hypermetrope as from the physical centre of rotation of the eye of the emmetrope. But the emmetrope alone will perceive this object side perspective, because for the ametropes it will be altered by their spectacle lenses (according to the ratio of the tangents), and they receive the identical perspective of the object space under altered angles. It will be understood that, therefore, the perception of space will be affected, and from what was stated on page 31 it is clear that for a known object the relief for myopes might be deepened and for hypermetropes flattened. (The other effect, lowering or raising the background, is not so important for spectacle wearers.)

From this discussion on the alteration in direction caused by spectacle lenses, it will be under-

stood why, for example, cataract patients, with their correction, do not reach out far enough to touch the object desired, or complain of a (relative) enlargement of the background. In high myopes, if the correction be worn, the opposite may be noticed. These effects gradually disappear as the person becomes accustomed to the laws under which the spectacle transmits to him the directions of the outer world.

The Development of Lenticular Diverging Lenses.

The diminution of the angles of the principal rays on the object side by a diverging lens, and their increase by a collecting lens, has the effect, in the reproduction of detail, that a myope will miss small objects more easily than a hypermetrope, because to the latter they will appear under a larger angle, and may more easily call forth movements of the eye. Against this, however, equally large fields of vision on the eye side will, for the myope, correspond to a larger field of vision on the object side than for the hypermetrope; this being the advantage gained by the myope for the lowered acuity for detail and the price paid for it by the hypermetrope. The boundary of the field of vision on the eye side is formed by the edge of the glass, which may be either circular or elliptical.

It will therefore be possible in diverging lenses to select a smaller size lens, and still have a sufficiently large field of vision on the object side. This is particularly advantageous in diverging lenses of high refractive power, because in these it is, more especially, the thick peripheral part which adds so much to the weight of the whole glass. In order to avoid the conspicuous appearance of very small lenses, it is possible to use lenses of ordinary size in which the peripheral portion has been altered into a light carrying border, of which many forms

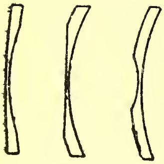


FIG. 34. The carrying border in dispersing lenticulars.

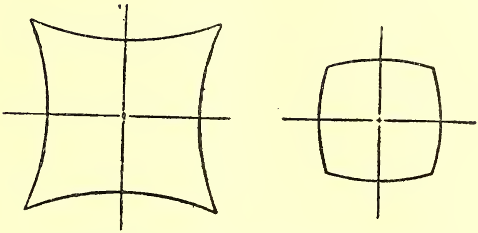


FIG. 35. The distortion effect caused by
converging diverging
lenses.

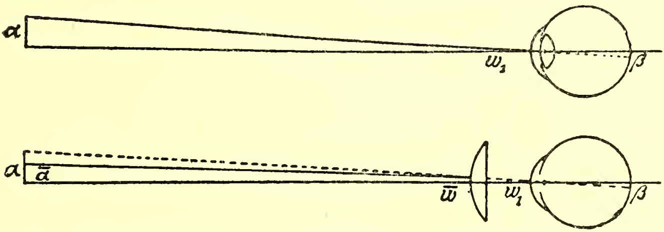


FIG. 36. The magnification of the retinal image in a corrected aphakic eye.

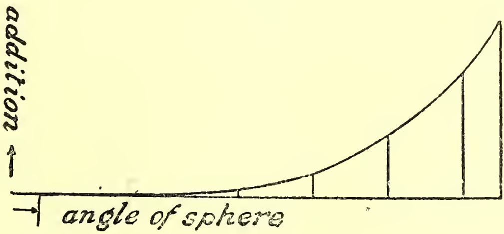


FIG. 37. An exaggerated representation of the material added to the vertex sphere.

have been proposed, as may be seen from the sketches in fig. 34. They are known as lenticulars. If, as will be seen later, this were attempted in converging lenses of high refractive power, a very great decrease of the field of vision on the object side would result.

Distortion Produced by Exactly Reproducing Lenses of Medium Thickness.

The subject of the alterations in direction on the eye side cannot be left without at least mentioning distortion, since the occurrence of this error in the image alters somewhat the results previously obtained. On carefully observing the image obtained from an exactly reproducing lens, it will be seen that the directions on the eye side are altered too much by the peripheral parts of the lens compared with the ideal case of a constant ratio of the tangents.

Converging lenses increase and dispersing lenses diminish too much the directions of the object side field of vision. This peculiarity, which is represented in fig. 35, is termed the distortion of lenses. It is most obvious when the object is a straight line which does not pass through the centre of the field of vision, for then it will appear curved, and the convexity of the curve in converging lenses and the concavity of the curve in diverging lenses will be turned towards the centre of the image. This error of the image will not influence the clearness of vision, but will only affect the direction of the object perceived. It may in general be regarded as an æsthetic error, and will not be perceived by the majority of wearers of glasses. It is not possible to correct this error in exactly reproducing spherical lenses of medium thickness, as above described, and we can only say that it is somewhat less marked in the Wollaston form than in the Ostwalt form.

Gullstrand Cataract Lenses with one Aspherical Surface.

In a previous section (page 60) mention was made of the limits within which exactly reproducing spherical lenses were effective. It will immediately be seen that the limit is only of importance in convex lenses. It is true that there is a limit for concave lenses, but this lies beyond −25 D, and therefore will only apply to lenses used for the correction of very high myopic axial refraction. In such degrees of myopia the correction of the astigmatism of oblique pencils, by means of a single spherical lens, will scarcely be of value, for the visual acuity of such eyes is almost invariably much reduced. But with convex lenses the case is quite different, because the error occurs in lenses as low as about +8 D. This will suffice for practically all cases of congenital hypermetropia, but the great class of aphakics will furnish a large contingent of people who will require convex lenses of even higher refractive power than +8 D. To this class will belong all those eyes which have healed up without any post operative astigmatism, and which before the operation had no axial refraction greater than −3 D.

The necessity of correcting the astigmatism of oblique pencils in these cases is all the greater, the less the originally good condition of the retina has been altered by the operation, because, as we know from experience, placing a converging lens of medium thickness in front of an aphakic eye causes a material increase in the size of the retinal image. In order to make this fact clear we may proceed in two ways.

It is not to be expected that an entire eye and a system made up of a convex lens and an aphakic eye, will have the same refractive power. As a matter of fact, they have not, and calculations with

the schematic eye show that the refractive power in the combination of aphakic eye and correcting lens is less, and therefore the size of the retinal image is greater. From which it can be directly concluded that the visual acuity will be increased according to the increase in the size of the retinal image. There is nothing to be said against the above method of representation, but it only demonstrates that the matter can be dealt with mathematically, and it applies the mathematical result without insight into the course of the calculation.

A clearer idea of the reason for the increased visual acuity may be obtained from the method shown by the Swedish ophthalmologist, K. Bjerke. For the sake of simplicity an emmetropic entire eye is assumed, and this is contrasted in fig. 36 with the performance of a corneal system with which alone we have to deal in aphakic eyes. There is a point, the optical centre of the crystalline lens, the ratio of the tangents of which (as far as exact measurements in the eye can be obtained) will not be altered at all by the operation. If we now consider a paraxial ray passing through it under the angle w' cutting the retina at an axial distance β, this ray will emerge from the cornea under the angle w_1, and will appear to come from a point 5.4 mm. distant from the cornea, the apparent centre of the crystalline lens. Thus far the course of the ray is the same in the entire eye and in the aphakic eye.

If, now, the course of the ray of the inclination w_1 in the entire eye be followed further backwards it will be found that a plane at right angles to the axis, say the test type, set up at a sufficient distance, say 5 m., will be cut at an axial distance α. But since the emmetropic eye is set for this distance, α and β will correspond as object and image, and $\frac{\beta}{\alpha}$ is the scale of the image upon the retina.

The course of the ray of the inclination w_1 in the aphakic eye under similar conditions may now be followed, the retina and the test type will be cut in the same places, but since the aphakic eye is not set for the test type it is not possible to speak of an image formation. In order to bring this about it is necessary to so place before the eye a convex lens of requisite refractive power $\overline{D}$ that there shall be a distance of 12 mm. from the inner surface of the lens to the summit of the cornea. This convex lens will inevitably reduce the angle w_1 to $\overline{w}$, and the ray of lesser inclination will necessarily cut the test type at a lesser axial distance $\overline{a}$. Therefore β and $\overline{a}$ will correspond as object and image in the spectacled aphakic eye, and the scale of the image on the retina is $\frac{\beta}{a}$, of which it is certain that

$$\frac{\beta}{\overline{a}} > \frac{\beta}{a}$$

and thus it is directly proven that the retinal image in the spectacled aphakic eye is enlarged.

If, then, the visual acuity of aphakic eyes be increased by ordnary cataract lenses, the correction of the astigmatism of oblique pencils in these lenses becomes of special importance: and the question arises; What methods are available for this purpose when bending the lens is not sufficient by itself? In any case this is not possible with single spherical lenses. The first method of overcoming the difficulty was put forward by the Parisian ophthalmologist, H. Parent. It consisted in replacing the single lens by two spherical lenses close to each other. It is possible by this method to correct the astigmatism of oblique pencils, but it will not be discussed further here because it is not a very practical

method. The frames for such a pair of lenses would be clumsy and expensive, and the durability, with ordinary care, very slight. The second possibility retains the single lens, but gives it one aspherical surface, and thus avoids the above mentioned difficulties in use. These lenses are known as Gullstrand's cataract lenses, after the discoverer.

The Nature of Aspherical Surfaces.

Before discussing the results obtained with Gullstrand's cataract lenses, it is necessary to refer to the nature of an aspherical surface. By this term is understood a surface of rotation, which differs from a definite spherical surface, the vertex sphere, in a regular manner. Since there are only slight differences between these two surfaces the terms spheroidal or deformed surfaces have been employed, being based upon the method of manufacture, but the term aspherical surface is better, because it is of more general application. For the purposes of this book it will suffice if we consider an aspherical surface to be formed by adding material to a spherical surface (the vertex sphere). This addition, as will be seen from fig. 37, is very slight near the vertex, but increases towards the edge, where it becomes quite marked, although the total quantity of added material is always very small. The result is that the curvature of the peripheral part becomes weaker for a convex surface and stronger for a concave surface. The alterations in curvature are very small in the centre but quite marked at the edge. As will be seen from figs. 38 and 39, in spite of the small quantity of added material there will result a finite alteration of the normal direction, the amount of which will increase the nearer the edge of the lens is approached. As an example, we may take the case of a cataract lens in which

$$A_{-25} = 12 \text{ D}.$$

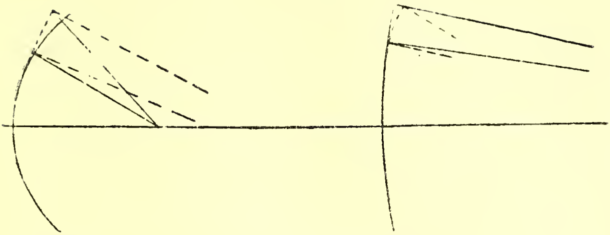


Fig. 38. Fig. 39.

The
flattening / deepening
of the vertex curvature by addition of material for a
converging / diverging
aspherical surface.

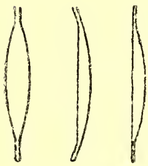


Fig. 40. The carrying border in collecting lenticulars.

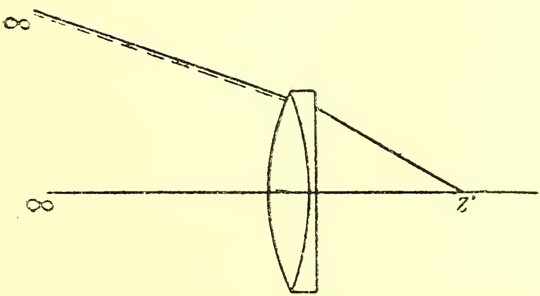


Fig. 41. The inclination of the coloured principal rays.

In such a lens the thickness of the material added to the edge of the inner concave surface will only be 160μ, while the radius of the vertex curve will be 140 mm. Obviously such a lens must be worked with extraordinary exactitude if the mathematical results are to be obtained in practice. At the edge of such a lens the alteration of the normal direction will be $2^\circ\ 8'$; this is no small amount, since the angle at the centre of the vertex sphere is only $6^\circ\ 52'$.

Gullstrand's Distance and Near Glasses for Cataract Patients.

Such an addition of material added to a surface, according to a definite law, is an extraordinarily efficient method of correction. By making convex lenses with such an aspherical surface it is possible to correct the astigmatism of oblique pencils even when the refractive power is far greater than +8 D, and there is no upper limit beyond which this correction cannot be applied, as far as lenses used in ordinary practice are concerned. It may be mentioned incidentally that the distortion can also be diminished by such asphero-spherical lenses. It is possible to remove it altogether, but in order to accomplish this it would be necessary to use extremely deep lenses so that they become very heavy and prominent. On this account one is content with merely diminishing the distortion. The curvature of the field of such an asphero-spherical lens can be calculated from the formula given on page 61; for most cataract glasses it is too small, as follows from the general laws. But this departure from the sphere (of virtual objects) which alone can be clearly seen by the moving aphakic eye is to a certain extent made harmless by the depth of definition.

In the upper section of the plate at the end of this volume in the columns marked *c* and *d*, there are

photographic reproductions of test letters, taken in green light. The column *c* represents letters as seen through a biconvex lens of +13 D, and the column *d* through a corresponding Gullstrand lens. In both cases the course of the rays are exactly such as occur in actual practice. As in the example previously given on page 60, the inclination of the rays on the eye side are also taken at 0°, 10°, 20°, and 30°, and it will be even more evident than in the case above given that the two lenses are of equal value only for rays passing along the axis. At an inclination of 10° the definition of the biconvex lens is much impaired, while with an inclination of 20° the recognition even of large test types has become impossible.

With reference to working and reading glasses for cataract patients, it will not suffice simply to take a lens of higher power, in which the astigmatism of oblique pencils has been corrected for an infinitely distant plane, but one must select a different system in which the astigmatism of oblique pencils has been corrected for objects at a definite working distance. According to the Strassburg ophthalmologist, E. Hertel, a suitable distance for such a reading glass would be about 25 cm. The reason for giving such a comparatively short working distance is because it is assumed that many aphakic eyes have only a low visual acuity, and therefore desire to see things as large as possible. In any case, whatever working distance is chosen it will only be necessary, in ordering the glasses, to mention such distance and the refractive power of the far lenses.

Lenticular Converging Lenses.

It will be easily understood that the weight of cataract glasses is of considerable importance. The axial thickness of converging lenses must become greater with increasing refractive power if the lens diameter remain the same, but the axial thickness

depends also on the shape of the lens, in such a way that the more strongly bent lenses are the thicker. The difficulties thus caused are not inconsiderable, and cataract glasses for distance, but more especially for near, may attain a considerable weight, and then if the frame be unsuitably chosen they will be intolerable to many noses. The most effective means of reducing this weight is to select a smaller lens, but the inevitable result of this is to diminish the field of vision. This latter, however, has not been the reason why this method has not been more widely employed. The real cause has been that of appearance, for cosmetic effect is of great importance to wearers of glasses. The result has been that the reduction of the lens diameter necessary to diminish the weight has been limited to the optically active central portion, and in order to obtain a lens of the ordinary shape and size a thin plane carrying border has been provided. In the old forms of glasses this could be done so much the more easily because the peripheral portion of the lens was of very little use for clear vision, on account of the astigmatism of oblique pencils. The shape of these lenses is very diagrammatically represented in section in fig. 40. At first they were made by cementing a small plano-convex lens upon a thin plane lens, but more recently they have been ground out of one piece of glass. They are known as Lenticular lenses.

In this connection it must be remembered that the conditions in the Gullstrand lenses are quite different; any diminution of the optically active part of the glass reduces by so much the perfectly useful part of the field of vision. Therefore, although it is possible to produce these lenses with a carrying border (*i.e.* in the form of lenticular lenses), these are not to be recommended. It is better to supply a suitable frame with a much broader bridge, so

that the nose will be able to bear the increased weight, and thus give the wearer the advantage of the large, clear field of vision. This will be discussed more fully when dealing with frames. The reduction of the field of vision to a smaller angle than is rendered necessary by the state of correction as regards the oblique pencils, is not to be recommended for converging lenses of high refractive power. As every strong convex lens has only a small field on the object side it is not to the advantage of the wearer to allow this amount to be further reduced for purely external reasons. It is quite possible to make a bridge of such shape as will enable a cataract lens to be worn comfortably, although the diameter of the optically active portion approaches closely to that usually given to lenses of low refractive power.

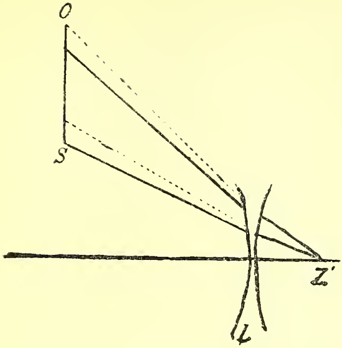
Spectacle Lenses Corrected for Chromatism.

An opportunity may now be taken to discuss the colour errors in spectacle lenses. An explanation of this point will be necessary, because nobody will have seen an object coloured when looking through a properly fitted trial frame or spectacle, although according to the theory of optics the different coloured rays from an infinitely distant axial point would be focused at the various colour foci of the lens. The reason why this error is not noticeable is because the eye is not a perfect optical instrument, but is itself the seat of chromatic aberrations; or, to use an optical term, the eye is a chromatically uncorrected system. Since the refractive power of the eye is 58.64 D, which is far more than any possible spectacle lens, and its chromatic aberrations are much greater than those of spectacle lenses, it would be quite useless to order any kind of achromatic spectacles in the ordinary sense. In this connection

the Swedish physicist, A. Gullstrand, has shown the way to an advance. He insists on the necessity of removing the difference of chromatic inclinations of the principal rays. It is necessary to so construct the system that the same object from the centre of rotation of the eye shall appear under the same angle w' for all the coloured principal rays. It will be easily understood why this must be so. If we imagine the freely moving eye seeing all around, it will be obvious that even in the most extreme outward movement of the line of vision no other chromatic errors of narrow pencils can occur than those that are present in the resting eye, namely, the chromatic aberrations existing along the optic axis. But with any lateral movement the position of the optic axis, *i.e.* the direction of the principal ray, is constant for all colours, because the whole system of the eye is turned about its centre of rotation. Thus the above-mentioned necessity in the chromatic correction of lenses is explained, namely, that in lateral visual directions the extreme* coloured principal rays, which have to be practically considered, must show the same direction. These two coloured principal rays must pass through the same point in the object space, *i.e.* they must emerge parallel to each other, when the object, as with correcting lenses, is at a great distance. This condition can be fulfilled by a system made up of two components cemented together. Such a system can be of the form represented in fig. 41.

Contrasted with such chromatically corrected systems are the ordinary spectacle lenses which, as will be seen from figs. 42 and 43, show lateral objects to the moving eye, with a coloured margin,

* It has become customary in technical optics to term extreme colours those which have to be considered in instruments for visual purposes, namely, those colours which correspond to the Fraunhofer lines C (red) and F (blue).



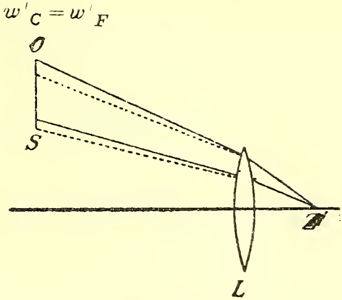


FIG. 42. FIG. 43.

Diagrammatic representation of the coloured margins of a lateral black object SO upon a white ground for a

diverging lens — converging lens

and the (——) red and (......) blue rays on the object side.

The margins towards the axis are

red — blue

and the margins away from the axis are

blue — red.

FIG. 44. Telescopic spectacle for high myopia.

the arrangement of the colours in converging lenses being the reverse of that in diverging lenses. It will be clear that for lenses of high refractive power these coloured margins will be more obvious than in lenses of low power, and this is one reason why no chromatic correction is attempted in ordinary spectacle lenses. The principal reason, however, is the great increase in weight necessary to produce a system, made up of two different lenses, of opposite sign, cemented together. In lenses of very high refractive power, as, for instance, cataract lenses, for the use of eyes with good visual acuity, chromatically corrected systems would be of considerable use; but even in such a case, in the interest of least possible weight, one will generally be content with the single Gullstrand lens.

The Telescopic Spectacles.

If now we proceed to deal with the more complicated forms met with in spectacle optics, mention must be made of spectacles constructed with two separated systems, namely, telescopic spectacles. As with simple lenses these may be divided into correcting lenses, and presbyopic and loupe lenses. Here also the most important form is the correcting telescopic lens, with which we will now deal.

Correcting telescopic spectacles were first proposed for high myopic patients, towards the end of the eighteenth century, but were not much adopted. In recent times the problem was raised by the Strassburg ophthalmologist, E. Hertel, and worked out by Messrs. Zeiss. The problem was to give high myopic eyes, by means of a spectacle, vision as good as could be obtained by the successful performance of Fukala's operation of removal of the crystalline lens. The increase in the size of the retinal image results from the peculiar situation which may be given to the principal points of the

telescopic spectacle by a proper arrangement of the component parts. The vertex refraction A_∞ and the refractive power R_{12} of the telescopic lens are quite different. In explanation it may be said that in the case of fig. 17, on page 39,

$$A_\infty = -18.2 \text{ D},$$

but

$$R_{12} = -11.4 \text{ D},$$

and the posterior principal point H'_{12} of the lens will be situated 19.2 mm. behind the anterior principal point of the eye H. There is thus an entirely different arrangement of these important points to that found in thin lenses. The result, which interests us here, being, that there is a very definite increase in the size of the retinal image. According to the lesser or greater refractive power of the component parts of such a system, one can obtain an increase in the size of the retinal image in round figures, of 30, 50, or 80%. One will scarcely ever be able to go higher than 100%, that is a magnification of two, on account of the accompanying increase of weight. Of course, the appearance of such a spectacle, as will be seen from fig. 44, is very different from the thin lenses usually employed, and there is very little probability that such appliances will ever be made so as to be less conspicuous.

In working out the above problem the chief difficulty was not merely in so arranging the component parts as to give a definite increase in the size of the retinal image, but in correcting the astigmatism of oblique pencils. Up till recently proper attention has not been directed to this most important point, as may be gathered from the fact that many proposals have been made to give the component parts of such a system a variable separation. However convenient such an arrangement might be in making it adaptable to the

ametropia of any individual, it is quite incompatible with the much more important requirement of exact reproduction over the whole field of vision. It becomes necessary, therefore, to maintain a fixed distance between the two component lenses. Under these circumstances it becomes possible by suitably bending both lenses to correct the astigmatism of oblique pencils for a finite angle of rotation w'. The images along principal rays of lesser inclination, however, will not necessarily also be free from astigmatism of oblique pencils. We may find astigmatic zones if we select too large an angle as the extreme inclination.

The same method is used to represent the astigmatism in this case, as was employed when discussing this error in single thin lenses. But owing to the construction of the telescopic spectacle out of two systems of high refractive power, the angles of incidence and refraction are of considerable size, and therefore the astigmatism of oblique bundles is not necessarily corrected for all intermediate visual directions when it has disappeared for a certain extreme angle. We shall consider two cases, of which the first will have very little, and the second considerable astigmatic zones. In the first case, in fig. 45, the magnification of the retinal image is 1.3 times, and comparatively low refractive powers of the component lenses will suffice; but in the second case, as in fig. 46, there is a greater magnification, namely, 1.8 times, and the refractive powers of the component lenses are much higher, and correspondingly the zonular astigmatism of oblique bundles is much more marked.

If the field of vision be selected of such size as to render the astigmatic zones not noticeable, then the extent of the eye side field will vary, in inverse ratio, with the magnifying power of the telescopic lens, that is the greater the magnifying power the smaller

FIG. 45.

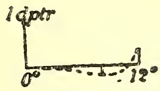


FIG. 46.

The astigmatic zones in telescopic spectacles.

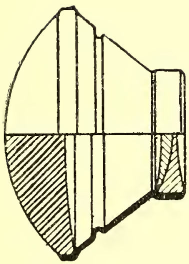


FIG. 47. A correcting telescopic spectacle for moderate ametropia, in about natural size.

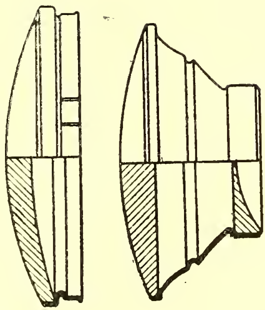


FIG. 48. Additional lens *a* for a correcting telescopic spectacle, *b* for a high myope. Approximately natural size.

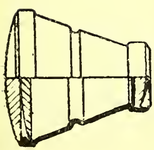


FIG. 49. A loupe spectacle formed of two component systems. Approximately natural size.

will be the field. This relationship has already long been known to exist in similar instruments, *e.g.*, the ordinary Gallilean telescopes. In the case of the telescopic spectacle a field of 40° can be obtained with a magnifying power of 1.3, but in order to obtain a magnification of 1.8 this field must be reduced to 24°. It may be mentioned that fields of vision of this extent can be obtained without making the whole spectacle too heavy to be worn in a properly adapted frame, such as will be described later, on page 118.

The correction of oblique pencils in telescopic lenses is not limited to their astigmatism, but extends also to their distortion, and it is possible to correct this error as well as the astigmatism of oblique pencils. In addition to this it is also possible to counteract the different inclination of the principal coloured rays. It may be said, therefore, that these greater optical means, the two separate lenses and the distance between them, have been made to serve their utmost purpose. The hope may be expressed that highly myopic patients will find great help from this apparatus, provided always that the changes in the fundus have not progressed too far.

Telescopic Spectacles for Eyes with Low Ametropia.

This opportunity may be taken to remark that recently, upon the request of several people, telescopic spectacles have been made for people with low degrees of ametropia, either for constant or occasional use, as, for instance, at the theatre or in galleries. In these cases it is also necessary for the wearer to have his ametropia accurately measured. The smaller this error the nearer will the spectacle approximate the ordinary Gallilean telescope with

a magnification of 1.8. For an emmetrope it will only differ from it in the method of its construction, and this will be in order to diminish the weight as far as possible. For this reason the additional lens, employed for the purpose of correcting the difference in inclination of the principal coloured rays, is not applied to the large converging lens, but to the smaller diverging lens which is nearer the eye. The strong curvatures which this procedure renders necessary is not so detrimental as it would be in the Gallilean telescope, because the spectacle frame can be so arranged that the centre of rotation of the eye shall occupy the correct position in the optical system.

It is imperative in fitting such a telescopic spectacle with a magnification of almost double, that the centre of rotation of the eye of the wearer shall actually occupy the position which was assumed in the calculation; in other words, it must lie centrally and at a distance of 25 mm. from the vertex of the lens nearest the eye. This condition, as was explained on page 55, will give a distance of 12 mm. between the surface of the lens nearest the eye and the summit of the cornea, when the eye is looking straight forward.

The accompanying fig. 47 will show the construction of such a system for an emmetrope. In the whole of the eye side field of vision of about 30°, which this system possesses, there will be no astigmatism of oblique pencils, no distortion, and no troublesome colour errors. It will be noticed that the curvatures, especially of the diverging lens, are fairly high, and hence arises the necessity of carefully considering the position of the centre of rotation of the eye. By this means it is possible to produce very light systems with a magnification of about double, and having a comparatively large field of vision; but the diminution in weight has to

be paid for by a greatly increased exactness in fitting them.

On account of the appearance such telescopic systems for low ametropias may be made not only as spectacles but also as a lorgnon. In the latter case there must be sufficient supporting surface, in order that the glasses may occupy correct positions relative to the centres of rotation of the eye.

Telescopic Spectacles for Presbyopes.

It will only be possible to use correcting telescopic spectacles for reading in the case of young people, and then only with glasses of low magnifying power; for the alteration in the range of accommodation produced by telescopic spectacles is much greater than that produced by the ordinary thin lenses; and this alteration is always in the direction of an increase of accommodative effort. For the presbyope it will therefore be necessary to employ special telescopic presbyopic glasses, and for these the object distance s, and the vertex refraction of the lens surfaces A_s are given.

It is, of course, quite possible to calculate a presbyopic glass for any given value of A_s, if the magnification of the retinal image be given. This magnification may be anything below 1.8 to 2.0, but it must be understood that the greater the magnification the smaller will be the diameter of the field of vision on the object side. The reasons for this are the same as were previously given for the decrease of the field of vision on the eye side. It may be mentioned that it will be advantageous for the patient to use his accommodation, even when wearing a presbyopic telescopic spectacle, and he can thus use quite a large proportion of his available amplitude of accommodation.

It will, however, be found to be more generally useful to employ an additional lens to the distance

glass for near work. It is only necessary to remember that the function of the additional lens is to reproduce the working surface at an infinitely distant plane, and that it then becomes visible by means of the distance correction. This applies also to the telescopic spectacle, and this method will be employed all the more readily as there will be a marked decrease in the cost, because an additional lens is much cheaper than a complete presbyopic telescopic spectacle, and by means of two different additional lenses of suitable refractive power it will be possible to employ the correcting telescopic spectacle for two different near points, as, for instance, for reading and for piano-playing. As a rule, the additional lens will be placed in front of the converging lens, but it is conceivable that there may be cases where it will be better to place such an additional glass behind the diverging ocular lens.

Loupe Spectacles Consisting of Two Components of Opposite Sign.

It is possible to use a similar system composed of two parts of opposite sign for the construction of a loupe or magnifying lens. Such systems have the advantage of a long free distance; and by a suitable arrangement of the parts it is possible to construct a binocular spectacle of moderate magnifying power (two to three times) with a working distance of 25 cm.; such an instrument might be of use for ophthalmologists. This matter really belongs to the work rather of the designer of loupes than of spectacles, and as a matter of fact such loupes with a long object distance have been known since the forties of last century under the name of Chevalier-Brücke loupes.

They are likewise constructed with two component systems, a converging object lens, and a diverging ocular lens, but these were mostly hand

or stand loupes. It was practically impossible to construct such a system, which could be worn like a pair of spectacles, as long as it was thought necessary to retain the achromatic objective, for then the weight was too great. It is only since Gullstrand showed the chromatic correction to be sufficient after removing the chromatic difference of the inclination of the principal rays, that it has been possible to use non-achromatic converging systems, and to transfer the whole chromatic correction to the light negative lens, as is represented in fig. 49. The exactly reproduced field of vision of such a loupe system will attain, even with a magnification of three times, a diameter sufficient for all the purposes required by the ophthalmologist.

Once again we must emphasise the necessity for care in exactly fitting such loupes to the wearer. The system of construction can be seen in fig. 49. The positive objective consists of two closely approximated single lenses, and the negative ocular is made up of a crown and flint lens cemented together so as to correct the colour aberration of the principal rays.

Telescopic Loupes.

Patients who require a still greater magnification of the retinal image must be provided with apparatus of quite different construction; although, on account of their weight, it is not possible to wear them as spectacles. Such systems are based on the principle of the prismatic telescope, having a magnification of four to six diameters. The short length and comparatively light weight of these instruments make them adaptable for the purpose required. By adding to the eye-piece a spectacle lens of such power as will correct the ametropia of the observer, such a telescope can be employed by these patients for looking at distant objects; and,

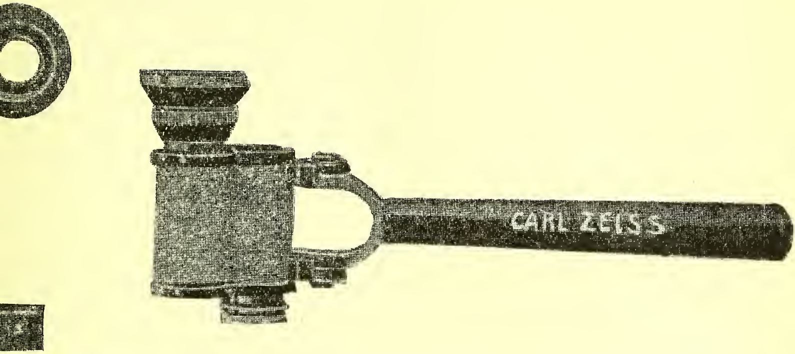


FIG. 50. A telescopic loupe with an additional lens for the objective of +3 D and one for the eyepiece of −15 D.

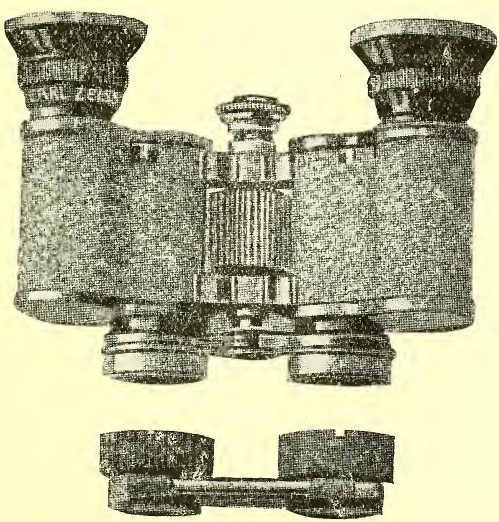


FIG. 51. A binocular telescopic loupe consisting of a binocular telescope of short inter-objective distance and additional lenses.

further, by adding to the objective a lens corresponding to the working distance, the patient will be enabled to see near objects through this strongly magnifying system. As a rule, one will have to deal only with monocular instruments, but there is no reason why they should not be made in the form of binocular telescopic loupes, and thus be available for both eyes of patients with low visual acuity.

Bifocal Glasses.

Bifocal glasses have the same object as the additional reading lens, namely, to enable presbyopes to see with ease both for distance and near. They are almost exclusively used nowadays for this purpose, for they are very convenient and can be made in a form in which they are scarcely noticeable. This increased convenience of bifocal lenses is obtained by giving the upper portion of the glass the refractive power D_d requisite for distant vision, and the lower portion the refractive power D_n, requisite for near vision. There will therefore be a difference in refractive power between the two portions of several diopters, which, as a rule, since

$$D_d - D_n \leqq 4 \text{ D},$$

will be generally less than four diopters.

This is one of the peculiarities of all bifocal lenses which, in other directions, show many differences. In order to obtain a general idea of the various possibilities, one must remember that there are two conditions which are opposed to each other, and between which one has to decide. Theoretically there ought to be a gradual transition from the distance to the near portion; while good appearance requires that the converging effect of the near portion shall be as inconspicuous as possible. Bifocal lenses may be made by mechanical combination, cementing, cutting out, grinding, and fusing. The

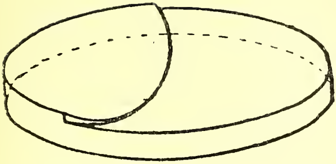


FIG. 52. A concave bifocal lens with the point of fusion of the two bounding surfaces.

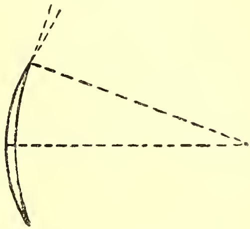


FIG. 53. The pair of tangents at any boundary point of an inconspicuous bifocal lens.

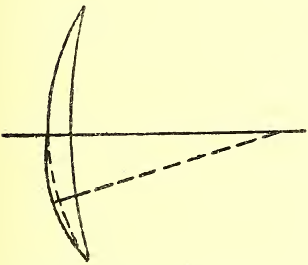


FIG. 54. Section through an inconspicuous bifocal lens, having a large distance portion.

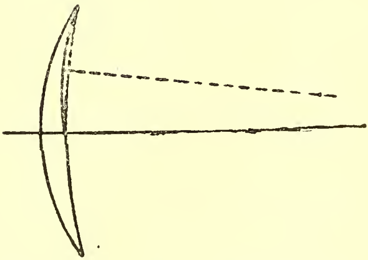


FIG. 55. Section through an inconspicuous bifocal lens, having a small distance portion.

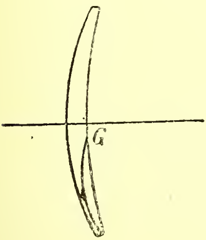


FIG. 56. Section through a fused bifocal lens, having a large distance portion.

first bifocal lenses were constructed in 1784 by Benjamin Franklin. They were, especially in the beginning, very little used, although they were never forgotten. Of late years, more especially in America, bifocal glasses of various kinds have been introduced and widely used. Of the various methods of manufacture the only ones much employed at present are grinding and fusing.

If the bounding surfaces are assumed to be spherical, the following general statement can be made. Since a gradual transition of two surfaces into one another—and with this a gradual transition of the two images into each other—can only take place where they have a common tangent, and since two spheres can only touch each other at one point, it follows, as will be seen from fig. 52, that for two different spherical surfaces which coincide with each other at one point, at all other points there must be a finite difference in height. Or, in other words, ground bifocal lenses with spherical surfaces are not inconspicuous when there is at one point a gradual transition from the distant to the near image. If, however, the two surfaces are so arranged that at no point is there any difference in height there will exist, as will be seen from fig. 53, at every point of the boundary line a pair of tangents with a finite difference of direction, *i.e.* there will be a sudden transition from one image to the other. Or, in other words, ground bifocal lenses with spherical surfaces show a sudden transition from one image to the other when they are made so as to be inconspicuous.

The difference in refractive power can be produced in one of two ways: either the distance glass can be taken as a basis, as is usually done. For instance, in the Uni-bifo and Uni-bifo luxe lenses, the near portion with its increased refractive power has, so to speak, to be ground on to the lower portion of the lens, as is shown in fig. 54. This entails

a troublesome and expensive process. Such glasses will always serve their purpose well for people whose chief occupation is out of doors and who only occasionally require to use the near portion. But if the wearer is one whose occupation is chiefly near work, and especially if he have a lessened accommodative power, then it will be better to proceed by taking the near portion as a basis, as in fig. 55, and to add to the upper portion the requisite dispersing effect by means of grinding, which is a much simpler and cheaper process, so that when the wearer looks up he will use the smaller distance portion. In both cases it will be well to start with that form of Ostwalt lens which will do for the principal portion of the lens.

If we now proceed to deal with fused bifocal lenses, it will be seen that the whole of both surfaces are worked to a uniform curve, as in fig. 56, while the additional refractive power is obtained by a converging lens of higher index of refraction—generally a flint lens. In the form which has become most widely known, the Kryptok, this near portion is introduced eccentrically in the lower half of the lens, and there is therefore a sudden transition from one image to the other at the boundary line, where there is a prismatic effect. These glasses are quite inconspicuous, and many different methods of manufacture have been proposed, but none of them have succeeded in giving an ideally perfect inner surface. Apart from this disadvantage, another also exists, namely, the introduction of chromatic errors into convergent lenses which is produced by the employment of a concave lens of crown glass and a convex lens of flint glass to form one system. These errors are not more obvious because the extent of the near field of vision is only small. The lenses produced by grinding will, on the whole, be more generally useful and also their weight will be less.

Prismatic Spectacles.

Since prismatic spectacles are not often ordered they may be dealt with here very cursorily. Prismatic glasses are ordered to correct errors in position (squinting) of the eyes; and therefore it becomes necessary to alter the direction of the rays, say those along the optic axis. The means employed for this purpose is the prism, and in general the prism will not have plane but spherical surfaces. It is important to remember that we are dealing here with a system having only one symmetrical plane. This plane will be horizontal when the deviation of the optic axis from its original position is in the horizontal plane.

There are several methods of measuring the abnormal direction. The oldest, but scientifically the least developed, is by means of the refracting angle of the correcting prism. Such a measure is, however, insufficient, since the index of refraction of the glass comes into play. If, for instance, we wish to obtain a deviation of 5° the refracting angle α of the prism will vary according to the index of refraction μ of the glass, or, more accurately, for

$$\mu = 1.52 \qquad \alpha = 9.6^\circ$$
$$\mu = 1.57 \qquad \alpha = 8.7^\circ$$

Since the deviating angle alone is of importance to the wearer it will be most practical if that be measured directly. For this purpose one may use the degrees of a circle, but it is better to use the arc of a circle, as is generally done in mathematics, because then two different systems of measurement (centradians and prism diopters) will coincide with each other with sufficient exactitude for all ordinary purposes. It is necessary to deduce the measurement of arc from the measurement of degrees.

If we take, as in fig. 57, the angle ϕ°, measured in degrees as the central angle of a circle, with

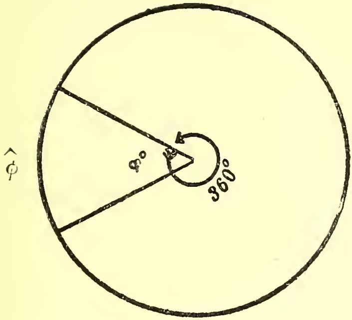


FIG. 57. To convert degrees into arc values.

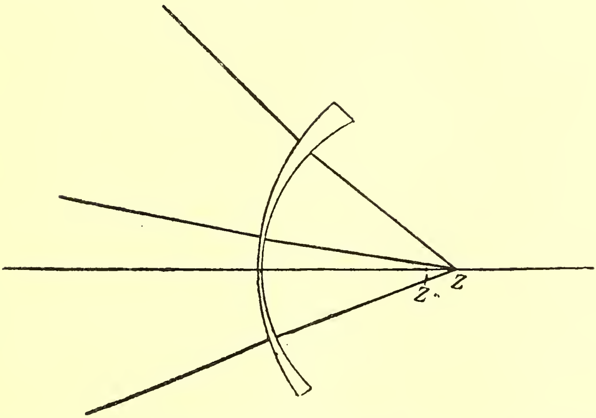


FIG. 58. An exactly reproducing prismatic lens of − 6 D. and 5.5 △.

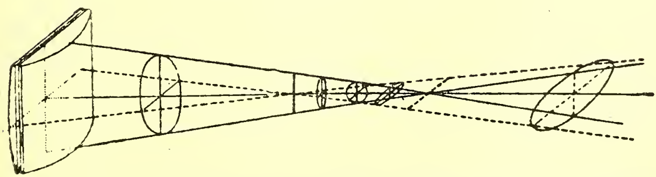


FIG. 59. A schematic representation (exaggerated) of a doubly symmetrical astigmatic pencil of rays.

radius $r = 1$, it will correspond to a certain arc of the circle $\hat{\phi}$, and the relation between the two is

$$\frac{\hat{\phi}}{\phi^\circ} = \frac{2\pi}{360^\circ} = \frac{\pi}{180^\circ}$$

because the length of the arc is proportional to the central angle; from the above we deduce the formula

$$\hat{\phi} = \frac{\pi}{180^\circ}\,\phi^\circ, = \frac{1}{57.296^\circ}\,\phi^\circ$$

which gives the relationship between the measurements of arc and degrees. The unit of arc measurement corresponds to the angle

$$\phi^\circ = \frac{180^\circ}{\pi} = 57.296^\circ,$$

and this is termed a radian.

Since this quantity is too large for the purposes of spectacle work, it was decided in 1891, upon the proposal of Dennett, to take as the unit of measurement for prismatic effect the hundredth part of the radian, which was called Centradian or Centrad ($\triangle$). The prismatic effect of one centrad being

$$1\ \triangle = 0.573^\circ = 34'\ 23''.$$

As has already been stated, the $\triangle$ value may, for practical purposes, be taken as equal to the prism diopter value, because we will scarcely ever have to deal with a prismatic effect greater than $6\ \triangle$.

The usual method of obtaining any requisite prismatic effect is to decentre the centre of rotation of the eye from the axis of the symmetrical lens. For this purpose Prentice's rule may be applied: for every 1 cm. of decentration from the axis a lens will have a prismatic effect of as many centrads as there are diopters of refraction.

It will be seen therefore that a prismatic lens will practically always transmit oblique rays; it cannot, therefore, be free from the astigmatism of oblique pencils, not even for the centre of the field of vision, and, as a matter of fact, most prismatic lenses suffer from this defect. If, however, the astigmatism of oblique bundles is corrected for this special direction, in other words, the prismatic lens is rendered anastigmatic, then we do not know how far this exact reproduction extends into the lateral field of view.

A very good method of making certain of obtaining a large angular value for the field of exact reproduction is to use an exactly reproducing lens of Wollaston form. These lenses have an extraordinarily large exact field of vision, and it is possible to cut out an eccentric portion of such a lens, as is shown in fig. 58, which will then have the requisite amount of prismatic effect, and be free from the astigmatism of oblique pencils. A further advantage of this method of procedure is that on account of the almost perpendicular path of the principal rays through the anterior surface, a layer of such a prismatic lens will be truly concentric to the centre of rotation of the eye, and therefore this lens will be as little conspicuous as possible.

In the third section of the plate, at the end of this book, are photographs, taken in green light, of the images formed by two prismatic lenses of $-6\text{ D} \supset 5.5\,\triangle$, in which the course of the rays is exactly the same as would occur in actual use. The columns *a* and *b* are those of a decentred biconcave lens, the columns *c* and *d* those of a glass cut out from an exactly reproducing Wollaston lens. The columns *a* and *c* represent the eye side inclinations within the plane of prismatic effect, and they lie upon the side of stronger deviation. The columns *b* and *d* represent eye side inclinations in a plane

at right angles to the preceding one. The columns *a* and *b* show an increasing error of focus with increasing inclination, but differ from each other; while the columns *c* and *d* show equally good results, whatever be the eye side inclination of the principal rays. Therefore we are justified in applying the term, exactly reproducing, to these lenses also.

ASTIGMATIC SPECTACLES.

The Astigmatism of the Eye.

We now have to deal somewhat more fully with astigmatism of the eye. The cause of this error of focusing is that one or more surfaces in the optical system of the eye are not spheres of rotation. For the sake of simplicity let us assume that the anterior surface of the cornea is such a surface, as, for instance, an ellipsoid of three axes. Then there will be no single image point which will correspond with the far point, even in the paraxial space; but, instead, the rays in the image space will form an astigmatic pencil. Such cases are termed corneal astigmatism, and this is the most common cause of this focusing error in the eye. Lenticular astigmatism does occur, but in ordinary observation it is taken together with the corneal astigmatism, and the whole effect is termed Total Astigmatism.

Let us next assume that these non-spherical surfaces of the eye are symmetrical to the optic axis; in which case it will be easy to construct a schematic representation of the course of the rays in the paraxial region, by placing two plano-cylindrical lenses of different refractive powers with their plane surfaces together, so that the cylindrical surfaces cut each other at a right angle. It will then be seen, as in fig. 59, that a similar astigmatic deformation arises, as has been pictured in fig. 22, page 48; the only difference being that in this case the con-

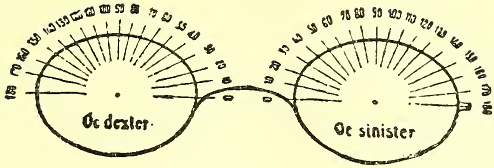


FIG. 60. The international scheme for indicating the axes of astigmatic lenses.

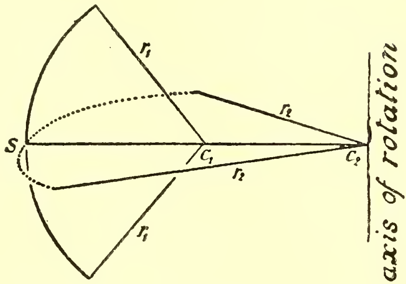


FIG. 61. Definition of a toric surface.

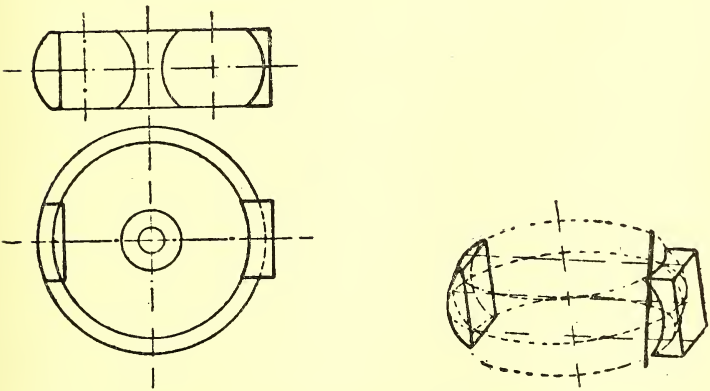


FIG. 62. A sausage-shaped toric surface.

Above: meridional section containing the axis of rotation. Below: section at right angles to the axis of rotation.

A perspective of plano-toric lenses of positive and negative effect.

ditions are simpler, inasmuch as the system now under discussion has two symmetrical planes—in the drawing a vertical and a horizontal one—and the principal ray is formed by the intersection of both symmetrical planes. These symmetrical planes are the principal planes of such an astigmatic system (page 47), with double symmetry.

There will therefore be, in the resting eye, two principal planes intersecting each other at right angles along the axis, and within which the focal lines, running at right angles to each other, will lie. If the object point of each principal plane be sought out, the focal line of which falls upon the fovea, one will obtain two different object distances $a_t a_{\int}$, as measured from the principal points H_t $H_{\int}$, the reciprocals of which,

$$\frac{1}{a_t} = A_t \, ; \, \frac{1}{a_{\int}} = A_{\int}$$

can be measured in diopters. The difference between them,

$$As = A_t - A_{\int}$$

is termed the Total Astigmatism of the Eye, and it may be remarked that there are very many cases in which the numerical value of this astigmatism is

$$As \leqq 4 \text{ D.}$$

Higher values of congenital error do occur, although more rarely. Occasionally one meets such values after cataract operations, and they are then regarded generally as cases of post operative astigmatism.

This, however, does not exhaust the subject of astigmatism of the eye. It is also essential to know the position in space which one of the two principal planes occupies; the position of the other will then also be determined, since it is always at right angles to the first. This position is given in degrees of arc which the axis of the correcting cylindrical lens will

occupy with reference to the horizontal meridian. There are many different methods of expressing this position in degrees of angle, and each method has some adherents. Fig. 60 gives the scheme adopted by the eleventh international Ophthalmological Congress, held at Naples in the year 1909. There is a separate scale for each eye, each of which is the mirror image of the other. This particular method was adopted because the positions of the axes in the two eyes, to the middle line of the face, are frequently symmetrical and thus can be expressed by the same numeral. Among English-speaking ophthalmologists the scale most frequently used is that known as the "Standard." In this the scales for the two eyes are the same, the zero point being on the right hand side of the oculist; thus, comparing it with the international scale given above, the notation for the right eye is the same in the two systems, while the "Standard" scale for the left eye is exactly the reverse of the international scale.

The astigmatism which arises along the optic axis of an astigmatic eye is due to a bi-symmetrical system, but the effect is the same as the astigmatism of oblique pencils already described. It is corrected by an astigmatic lens, namely, a system which is also bi-symmetrical. If a certain distance be assumed between the cornea and inner surface of the lens, it will be possible to determine the values δt δf, which are required, and thus to calculate, in the ordinary way (page 37), the vertex refractions $A_{\infty t}$ and $A_{\infty f}$, which are to be given to the symmetrical planes of the astigmatic spectacle lens, in order that the infinitely distant point shall be reproduced upon the retina without any astigmatism. Exactly the same calculations have to be made as for axially symmetrical lenses already dealt with on page 34; but the result will only apply to one prin-

cipal meridian. In this case, as also in the other, it is only the refractive power which is determined, while the shape of the meridional section of the lens can be varied.

The Usual Form of Astigmatic Spectacle Lenses.

The first method of making astigmatic lenses was by grinding the two powers upon one lens in the shape of crossed cylinders, each cylinder having one of the powers A_t and $A\int$. This, however, was soon abandoned in favour of the sphero-cylindrical form. In this form the spherical surface had the power A_t or $A\int$, and on the other side was ground a cylindrical surface of the power $-As$ or $+As$, so that the required refractive power was obtained:

$$A_t + O = A_t \,;\; A_t - As = A\int \text{ and}$$
$$A\int + O = A\int \,;\; As + A\int = A_t \,.$$

In the accompanying figures 64 and 65 such sphero-cylindrical lenses are represented through their symmetrical planes.

Finally the sphero-toroidal form was adopted, without, however, formulating any exact rules, as to when one or other form could be most advantageously employed. By a toroidal surface is meant a doubly symmetrical surface, which results, as in fig. 61, from the rotation of the sector of a circle about an axis which runs in the plane of the sector but not through its centre. It is therefore possible to differentiate between what may be termed sausage-shaped (fig. 62) and barrel-shaped (fig. 63) toric surfaces, according, as the axis of rotation lies farther from or nearer to the vertex than the centre of the circle. It becomes necessary to employ toric surfaces if it be desired to bend astigmatic lenses so as to produce results about as good as are obtained in the case of axially symmetrical lenses. Such bending of astigmatic glasses produces *exactly reproducing astigmatic lenses*. The reason for this

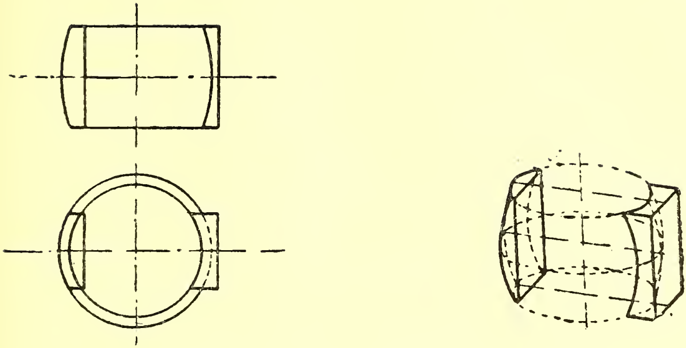


FIG. 63. A barrel-shaped toric surface.

Above: meridional section containing the axis of rotation.
Below: section at right angles to the axis of rotation.

A perspective of plano-toric lenses of positive and negative effect.

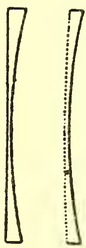


FIG. 64.

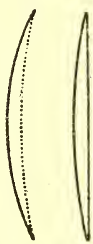


FIG. 65.

The symmetrical planes of a sphero-cylindrical lens of
−6, −4 D. +6, +4 D.
The dotted lines represent the arcs of rotation.

bending in axially symmetrical lenses is to overcome the astigmatism of oblique pencils, and thus to obtain in the peripheral parts of the field vision as clear an image as is obtained along the axis. The same reason applies to astigmatic glasses, and to obtain the same result, bending of the glass is resorted to in this case also. It must be remembered that a naked astigmatic eye never receives a clear image of any object upon the retina; and that by means of the correcting cylinder a clear image is only obtained upon the macula of the resting eye when the visual line coincides with the line of intersection of both symmetrical planes, which may be termed the axis of the astigmatic spectacle lens and is to be distinguished from the axis of the cylinder. Since, however, in ordinary use, the eye is being constantly rotated about its centre, which naturally does not affect the astigmatism of the eye, it is therefore requisite in an astigmatic glass that the astigmatism of principal rays of finite inclination shall always be equal in amount to the astigmatism along the axis of the lens, and that the situation of the two principal planes shall always be such as is required by an eye turned through a finite angle. The alteration of the situation in space of these principal meridians brought about by the movements of the eye has been carefully worked out by J. B. Listing, of Göttingen, and those requiring fuller information on this point are recommended to his works. Here we shall set the requirements for such a glass on a much lower scale.

Let it only be assumed that the visual line will move in the two symmetrical planes of the glass which coincide with the position of the principal meridians in the resting eye. The result of this simplified assumption is, that with such movements of the eye the plane of one of the two principal sections will not change at all, while the position of

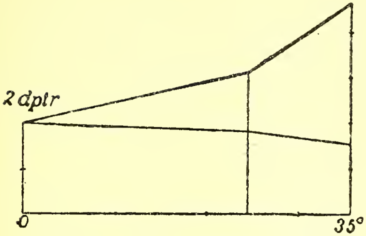


FIG. 67. +6, +4 D.

FIGS. 66 AND 67. The astigmatism of oblique pencils in the two principal meridians (——) and (——) for sphero-cylindrical lenses.

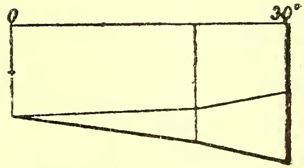


FIG. 66. −6, −4 D.

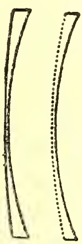


FIG. 68. FIG. 69.

The symmetrical planes of an exactly reproducing sphero toric lens.
−6, −4 D. +6, +4 D.
The dotted lines represent the arcs of rotation.

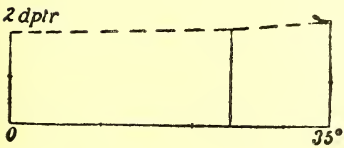


FIG. 71. +6, +4 D.

FIGS. 70 AND 71. The astigmatism of oblique pencils in the two principal meridians (— — —) for an exactly reproducing sphero-toric spectacle lens.

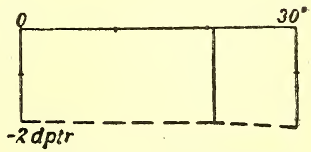


FIG. 70. −6, −4 D.

the other can be determined in a very simple manner, for it must always intersect the first one along the principal ray, and at a right angle. An astigmatic lens can thus be considered to be suitably bent and can be called *exactly reproducing* when, for any movement of the visual line in these two selected planes, the astigmatism of principal rays of finite inclination shall be the same as that along the axis of the lens. Experimental investigation has shown that in fulfilling these very limited requirements exact reproduction upon the retina is also obtained, even when the movements are such as to take the visual lines outside these two favoured planes.

Since an astigmatic lens has two symmetrical planes, it is only necessary to investigate the directions of the principal rays in the first symmetrical plane, say the vertical one, from the centre upwards, and in the second or horizontal plane from the centre to the right. In figs. 64 and 65 these symmetrical planes are indicated for sphero-cylindrical glasses.

Let it be assumed that a few of such principal rays are followed through the astigmatic lens in each of these two planes, and that by their means the astigmatism of oblique pencils is determined. It will then be found that in ordinary astigmatic spectacle lenses, say a sphero-cylinder, the astigmatism along principal rays of equal obliquity will differ in these two planes, and also it will differ from the requisite astigmatism as found along the axis. In figs. 66 and 67 the astigmatism in the first symmetrical plane is represented by a thick line, and in the second plane by a thin line. It will be at once obvious that the above statement was justified, for in one principal meridian the astigmatism diminishes towards the edge, while in the other it increases.

If we now compare with the above two exactly reproducing astigmatic lenses of equal refractive power and equal astigmatism, then in shape they will be somewhat bent, and they will possess a spherical and a toroidal surface. Sections of such lenses with their symmetrical planes are represented in figs. 68 and 69. If now, in this case, we also follow principal rays, limited to the two principal planes, through the lens, we shall find, as represented in figs. 70 and 71, certain variations from the astigmatism required along the axis, but they will be the same for the two principal meridians, and the amount will be very much less than in the preceding case.

In Group II. of the plate at the end of this volume the performance of the usual sphero-cylindrical lenses and of exactly reproducing sphero-toric lenses are contrasted. In both cases the photographs refer to the performance of a convex astigmatic lens, having a refractive power in the first principal meridian of +4 D, and in the second meridian +7 D. The photographs were taken with exactly the same direction of rays as would occur in the ordinary employment of an accurately fitted spectacle lens. They are taken with monochromatic green light, and with an inclination of the principal ray on the image side of 0°, 10°, 20°, and 30°. The columns *a* and *d* are photographs taken in the plane of the first principal meridian, the columns *b* and *e* in the plane of the second principal meridian, and finally the columns *c* and *f* in a third plane, which includes an angle of 45° with each of the two principal meridians. The columns *a*, *b* and *c* represent the performance of an ordinary sphero-cylindrical lens, and the columns *d*, *e* and *f* that of an exactly reproducing sphero-toric lens. It will be seen that for sphero-cylindrical lenses the performance, as shown in column *a*, in which the principal ray moves along

the first principal meridian, is alone bearable, although even here the definition becomes worse towards the edge. The columns *b* and *c*, in which the eye side principal ray moves in a different plane, show much greater diminution of clearness. The columns *d*, *e* and *f* show no signs of such diffusion, and therefore the term exactly reproducing applied to these sphero-toric lenses is fully justified.

All lenses destined to aid astigmatic eyes should be of such a correct shape, and the consideration of this correct shape would have to be discussed under the different headings already mentioned when dealing with axially symmetrical lenses. To deal with this subject fully here would make the book too large, but it is necessary to mention the subject in order to show that such considerations must be applied in dealing with astigmatic lenses. A few remarks, however, applying only to thin correcting lenses may be made.

There are various possibilities in calculating such a lens. It is possible to produce the cylindrical effect in one or other principal meridian, the cylindrical effect may be placed on the anterior or the posterior surface of the lens, and there is also the choice of the more or less strongly bent form. In the most favourable case there are eight different forms to choose from, but generally all theoretical possibilities are not available, and the choice is usually limited to four forms and sometimes only to two. The choice will usually fall upon that form which is least difficult to manufacture.

There are limits to the refractive power within which it is possible to produce exactly reproducing astigmatic lenses by simply bending them, in the same manner as these limits exist for axially symmetrical lenses. And it is only within these limits that curves can be found which will give the desired constancy of astigmatic effect for any movement of

the line of vision within the symmetrical planes. Should the refractive power exceed these limits recourse must be had in this case also to an aspherical surface of revolution. Since an astigmatic lens can only have one spherical surface, it is this surface which must be changed to an aspherical one. Especially will this be the case with cataract glasses, and in order to afford cataract patients, with post-operative astigmatism, equal clearness of vision in all parts of the field, it will be necessary to employ asphero-toric or Gullstrand's cataract lenses.

Spectacles as an Aid to Both Eyes.

In the first part of the book the subject of vision with two eyes was treated, and now a few remarks must be made as to the effect of a pair of spectacles on such vision.

It has already been mentioned that in employing a suitable glass for an eye, not only is the clearness of perception increased, but that an alteration in the direction in which an object is perceived also occurs.

In a pair of spectacles composed of two lenses there will be an alteration in direction for each of the two principal rays from any given object point. There will thus be two possibilities, either these two rays, on the eye side, if prolonged, will intersect, or they may cross each other.

Dealing with the first case will present no difficulty; each point of the object space will correspond to a definite image point in the eye space, namely, that point at which the principal ray directions would intersect if prolonged backwards. This condition will be quite or very approximately realised in the case of spectacles composed of two thin axially symmetrical lenses of equal refractive power. The deepening of relief for myopes and flattening for hypermetropes, which was mentioned on page 69, becomes much more obvious in vision with both eyes.

The second case, where the directions on the eye side cross each other, will not give rise to any single point in the eye space, and it is only possible for the wearer to have single perception when he causes his two visual lines to coincide with the directions, on the eye side, of the two principal rays, so that he will receive upon each of his maculæ an image of the fixed object. In such a case, however, the two visual lines will no longer lie in one plane, as was the case in vision with naked eyes; in other words, the wearer must overcome a vertical displacement caused by the glasses by means of an abnormal ocular movement.

The muscular apparatus of the two eyes can overcome, within certain limits, such a vertical error; and up to $1\frac{1}{2}^\circ$ this can be done without great difficulty. It would appear that the constant exercise of overcoming such a displacement as that caused by glasses, results in time in the muscles being able to overcome a greater amount of displacement than they would otherwise have been able to do.

The accompanying fig. 72 is a schematic representation of the course of two oblique principal rays, from one object point, both on the object side and eye side of the glasses. It will be seen that on the object side the two principal rays lie in one plane, that made up by the fixed object point and the two apparent centres of rotation (page 66), Z_l and Z_r. After passing through the lenses these two principal rays have been so altered in their directions that—if prolonged sufficiently far—they will cross each other. In fig. 72 this is indicated by the dotted lines, and the course of the left principal ray in the eye space is supposed to be below the right. If the cause of such a course be enquired into, it will be possible, from what has already been stated, to say that it could not occur in the case of axially symmetrical lenses of equal refractive power. It could, however, very easily occur in the case of

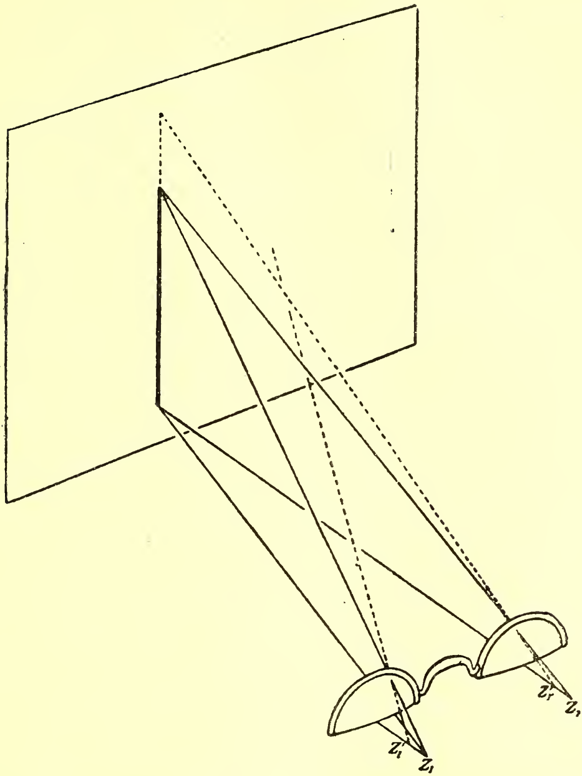


Fig. 72. The crossing of two corresponding visual lines on the eye side.

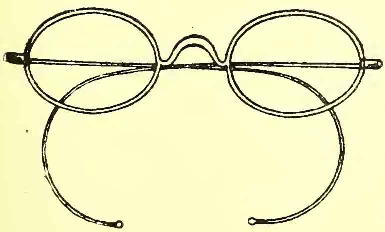


Fig. 73. A rim spectacle.

astigmatic lenses, where in certain lateral positions, the cylindrical effect of one glass could alter the direction of its principal ray in quite a different manner to the alteration produced by the other glass. This will also explain the difficulty which sometimes arises when people first wear astigmatic glasses, generally the difficulty soon passes off, but occasionally it persists, and thus prevents binocular vision.

Another possibility is that the two eyes may have a different ametropia (anisometropia). In such a case the directions on the eye side, even with axially symmetrical lenses, will be so different that they will cross each other for every object point out of the horizontal plane. In cases where the difference is small—as will be understood from what was said before—the difficulty will be easily overcome, and the wearers of such glasses will soon acquire binocular vision in spite of the difference between their glasses, but in cases of anisometropia of high degree (4 D and more) this will not always be possible. A case of special interest would be one where a cataract had been extracted from one eye only. Let it be assumed that the lens of one eye had suddenly become cataractous—say after an accident—and that this cataract had been successfully removed by operation, then these two eyes in spite of their healthy muscular apparatus, will have lost their ability to work together; because the resulting anisometropia of 11 D or more will cause such a difference in the direction of the principal rays on the eye side, that binocular vision will not be possible with spectacles of ordinary construction. It is possible to construct a complicated spectacle system which will enable such individuals with monocular aphakia to have binocular vision, but it is impossible to go into this question here.

III. SPECTACLE FRAMES.

The object of a spectacle frame is to place the lenses before the eyes in such a manner that the conditions of the calculation shall be fulfilled. To attain this object it is necessary that the centre of rotation of the eye shall be at the proper place, that is 25 mm. behind the posterior surface of the lens and upon its axis, and, further, that in prismatic lenses the one symmetrical plane, and in astigmatic lenses, the two symmetrical planes, shall occupy the requisite position in front of the eye.

For this purpose the most frequently employed are *spectacles, eyeglasses, or pince-nez,* lorgnons and lorgnettes, and these will now be considered.

Spectacles Proper.

One understands by spectacle an arrangement for holding lenses at a fixed distance from each other, resting chiefly upon the bridge of the nose and having some special arrangement for being held by the ears. It is very probable that the first spectacles proper were constructed in the beginning of the 18th century. There are two main kinds—rim spectacles and rimless spectacles.

In rim spectacles (fig. 73) the rim of the frame completely surrounds the lens, whether its form be elliptical, circular, or semi-circular. This form has the advantage that it makes a very firm support for the lens, and that the whole surface of the lens is available for visual purposes. The shape of the rim is generally oval (elliptical), but other forms occur such as circular, semi-circular and pantoscopic forms, of which the last (fig. 74) has a very large visual field, and yet is not too heavy or conspicuous.

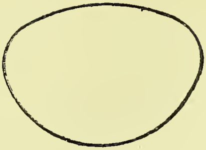


FIG. 74. Pantoscopic shape.

FIG. 75. A meridional section through a spectacle lens and frame.

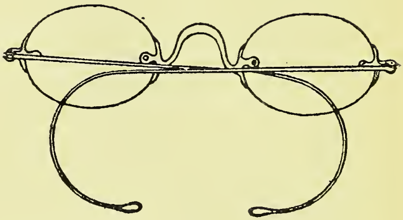


FIG. 76. A rimless spectacle.

FIG. 77. A bridge projecting inwards.

FIG. 78. A bridge projecting outwards.

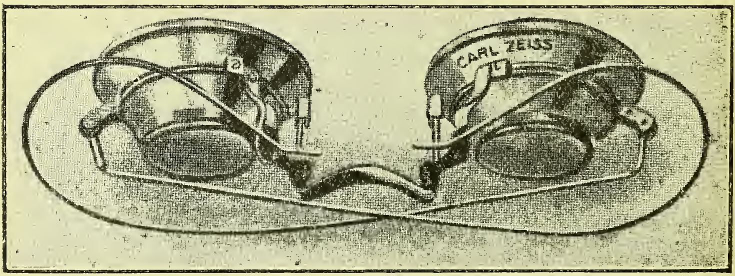


FIG. 79. Telescopic spectacle with plaquets.

The rim of the spectacle holds the lens, as a rule, by enveloping the somewhat sharpened, facetted edge of the glass, but there are forms where the rim is simply a wire which rests in a groove, in the edge of the glass. Oval lenses are so placed in the frame that with the long axis horizontal the plane of deviation in prismatic lenses, or the axis of the cylinder in astigmatic lenses, will occupy the prescribed position. It may be remarked that with oval lenses no extensive alteration of the position of the cylinder axis can be obtained; for this purpose the pantoscopic shape is more suitable, but best of all is the round form. Therefore one of these two latter forms should always be chosen when correcting a highly astigmatic eye, because in such a case it is most important to obtain an absolutely correct position for the axis of the cylinder.

Lenses for rimless spectacles (fig. 76) must be made somewhat larger than those meant to be placed in rims, because they are held firm by small screws at the nasal and temporal edges. The nasal screws may, in certain directions of vision, be projected upon the retina as very much diffused images, and may then interfere with vision. The lenses fastened by means of such screws are not held as firmly as those in rims—lenses in rimless spectacles work loose fairly easily—and quite often the glass breaks at the screw hole. In spite of these disadvantages, the rimless spectacle has been much used, as it is considered to look better, and therefore must be dealt with. They are sometimes described as being first made in Vienna. But whoever invented them, they must have been constructed before the end of the sixties of last century.

The principal materials used for spectacles are steel, nickel, gold, and rolled gold. By this latter is meant such a combination of base metal with gold, that the base metal inside is surrounded by a

thin covering of gold. As long as this latter remains unbroken, the spectacle has the appearance of a gold one. Such parts of the frame as are subject to special strain (*i.e.* the side wires) should have a thicker covering of gold than the other parts.

To the rims or end pieces are attached that part which rests upon the nose, the bridge, and the arrangement for holding on to the ears, the temples, or side wires.

The chief function of the bridge is to maintain the glasses in good position. In order that both lenses shall be properly centred, the bridge requires to be capable of alteration in two directions, horizontally and vertically. For this purpose bridges are supplied of different widths and different heights; for instance, the height of the bridge in fig. 73 is much less than that in fig. 76. In determining the style of bridge necessary it must be remembered that the eyes are frequently situated at a distance from the nose which differs on the two sides, and that the vertical situation may also differ on the two sides. As a rule the optician is able to make these necessary corrections by simply bending a frame which otherwise fits. Having thus placed the lenses so as to be correctly in front of the centres of rotation, it becomes necessary to make sure that these latter are placed at a correct distance along the axis of the lens. This will be the case when the distance between the posterior vertex of the lens and anterior vertex of the cornea is about 12 mm. This position can be attained by altering the projection of the bridge. In this connection the term projection inward is used when the line joining the bends in the frame passes in front of the bridge (fig. 77), and projection outward when this line passes behind the bridge (fig. 78). A further requisite is that the bearing surface of the bridge shall follow the contour of the nose as closely as

possible. In cases where specially heavy glasses are to be worn it will be better to remove the weight, either in part or entirely, from the bridge of the nose, and fix plaquets to the frame, which shall transfer the weight to the sides of the nose. Fig. 79 is an example of a frame of a telescopic spectacle, and the method of distributing the weight is well shown. These plaquets will be discussed more fully when dealing with pince-nez.

The sides may achieve their object in one of two ways, either by pressure on the head, straight sides, or by curling round the ears, curl sides. The first mentioned form are now comparatively seldom used, except for reading glasses, where it is necessary to take them off frequently. The curl sides have been of many forms before finally assuming their present form, which is that of a partly or entirely spun wire. This enables it to maintain the spectacle in good position without pressure of any kind if it be carefully fitted. The connection between the rims and the sides takes the form of a joint with vertical axis termed *the endpiece.*

Fig. 80 shows a frame drawn in perspective with the principal parts indicated. The bridge projects inwards and is of medium height.

A carefully fitted pair of spectacles requires a case for its protection. This should open completely, so that the spectacles can be easily put in and taken out, and when closed the case ought to exert no pressure upon the glasses.

The Pince-Nez (Eyeglasses).

Another possible method of correctly retaining framed or rimless glasses before the eyes is to have plaquets fitting the side of the nose and held there by the pressure of a spring. These are termed pince-nez or eyeglasses. Such a method of holding lenses is not as secure as the spectacle, for the

latter is supported at three widely separated points, while the former is supported at only two points, situated quite close to each other. Nevertheless, pince-nez do quite well for light lenses, and are much used, as they are supposed to have a better appearance.

Pince-nez may be divided into two main forms according as to whether the distance between the lenses depends upon the situation of the frame, or where this distance is fixed. The last method is the one now in more general use, and the one which is to be recommended.

Pince-Nez with Variable Lens Distances.

These have a spring above which presses the two sides against the nose. On account of the varying position which the lenses will take up, it does not appear advisable to use such a frame for astigmatic lenses. We shall have to deal later with pince-nez suitable for such lenses. But as far as axially symmetrical lenses of low refractive power are concerned, it may at once be said that the small variations in the position of the centres will not cause a great deal of trouble.

Those parts of the pince-nez which come in contact with the sides of the nose are called plaquets. The shape of the older styles, as in fig. 81, is that of a curve convex towards the nose, and permits of very little, if any, individual adaptation. It is obvious that the distance between the two plaquets depends upon the width of the nose, and therefore with wide separation between the eyes it is necessary to use very large lenses. Vertical adaptation can only be carried out to a small extent. A kind of bridge projection is possible by widening the plaquets towards the nose. As a rule the bearing surfaces of the plaquets are lined with celluloid, tortoise-shell, or cork, in order that the easily oxidised

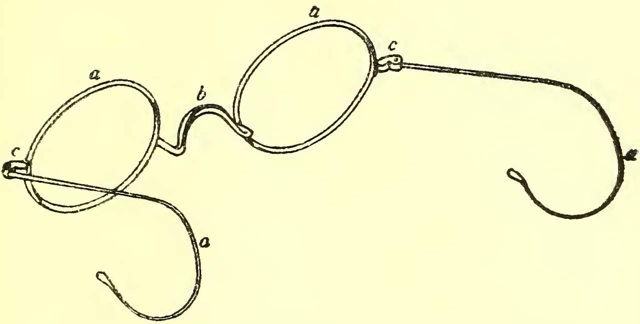


FIG. 80. Perspective drawing of a spectacle frame.

a a Eye wires.
b Bridge (projecting inwards and upwards).
c c End pieces.
d d sides.

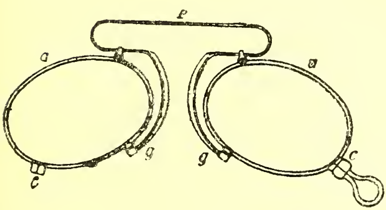
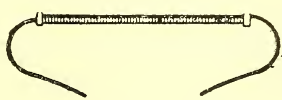


FIG. 81. Pince-nez with a variable distance between the lenses.

a a Eye wires.
c c Joints.
e Spring.
f Handle.
g g Plaquets.

FIG. 82. A rigid bar spring frame for astigmatic lenses.



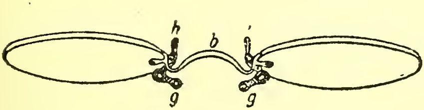


FIG. 83. Pince-nez with a fixed distance between the lenses.

b Bridge.
g g Plaquets.
h h Finger pieces.

metal shall not come in contact with the skin; but with gold or rolled-gold frames this is not necessary.

In the case of astigmatic lenses the old method was to use a frame in which the two rims were each attached to bars sliding on each other against a spiral spring (fig. 82). This method partly fulfilled the requirement that in astigmatic lenses the direction of the axis of the cylinder shall be independent of the position of the glasses. The spiral spring is made tense by pulling the lenses apart, and thus exercises pressure upon the nose. The plaquets of this form of pince-nez are usually attached to the frame by a joint with a horizontal axis, so that they may adapt themselves to the shape of the nose. They are called movable plaquets, but they do not entirely fulfil their object, and the distance between the centres of the lenses is not always the same, for this varies whenever the pince-nez is put on somewhat differently.

Pince-nez with a fixed distance between the lenses were first introduced by R. B. Finch, an American, in 1901. The method of construction, as in fig. 83, is to join the two rims by a rigid bar of metal, or bridge, the length of which can be varied according to the distance between the eyes. The bridge can be made of any required height, width, or projection, so that it is possible to obtain exact individual adaptation, by selecting a suitable bridge. It is also possible to obtain as correct a distance between the cornea and lens as the essential weakness of fit of any pince-nez will allow. It is obvious that such a pince-nez is equally well adapted for astigmatic as for axially symmetrical lenses, and it possesses great advantages over other forms. It is held in position against the sides of the nose by plaquets which are pressed inwards by small inconspicuous springs—and is put on and taken off by means of small finger-pieces acting on the springs. The

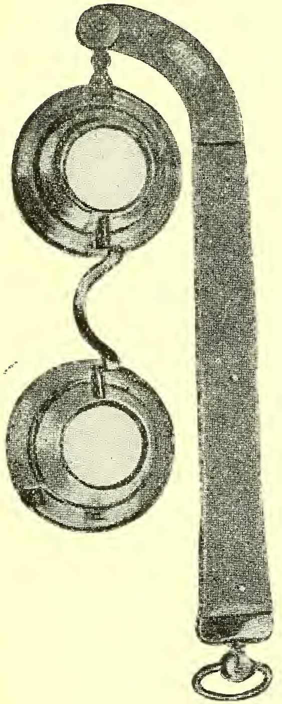


FIG. 84. A telescopic spectacle mounted as a lorgnon.

plaquets are easily bent so as to be adapted exactly to the nose of the wearer.

Pince-nez are best kept in a solid case, which will not exert any pressure upon its contents. A thing specially to be avoided in the old-fashioned pince-nez is the folding together, because it is then easily bent and the position of the lenses altered. The small hook and bar sometimes provided for this purpose should be invariably removed.

The *monocle* or single glass shall only be mentioned, since it is of little importance. It may be either rimless, with a smooth or rough edge, or it may have a rim which may be a rim only, or else provided with a gallery. The provision of a proper rim for a monocle is not a simple matter if it be necessary that the lens shall be used centrally.

Lorgnettes and Lorgnons.

These two forms of mounting are not meant for constant wear, but only occasionally held before the eyes. They are usually provided with a handle, and differ from each other in that in the lorgnette the lenses can be folded upon each other at a hinge in the bridge, while in the lorgnon the lenses are held in a rigid frame.

This form of mounting has been much used for two reasons: first, because it enabled the glasses to be rapidly put up and down, and, secondly, because it has always been regarded as very elegant. Recently this form of mounting has been used for carrying heavy systems, such as telescopic spectacles (fig. 84). In such a system it is essential that the centres of rotation of the eyes shall occupy exactly the position for which it has been calculated, and therefore these mountings have been provided with a bridge which can be altered so as to fit any individual and enable him to find easily the right position for holding the lorgnon.

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